Business Cycle Measurement

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1. Introduction

Introduction

- Before we go on to build models of aggregate economic activity that can explain why business cycles exist and what, if anything, should be done about them, we must understand the key features that we observe in economic data that define a business cycle.
- In this lecture, we examine the regularities in the relationships among aggregate economic variables as they fluctuate over time.

Deviations from trend versus cycles

- Typically, we think of a time series as the sum of four components:

$$\text{series} = \text{trend} + \text{cycle} + \text{seasonal} + \text{irregular}$$

- After removing the trend and the seasonal components, we are left with the business cycle and the irregular component. We refer to this as deviations from trend.
- For simplicity, in this lecture we neglect the irregular component and refer to deviations from trend as "cycles".

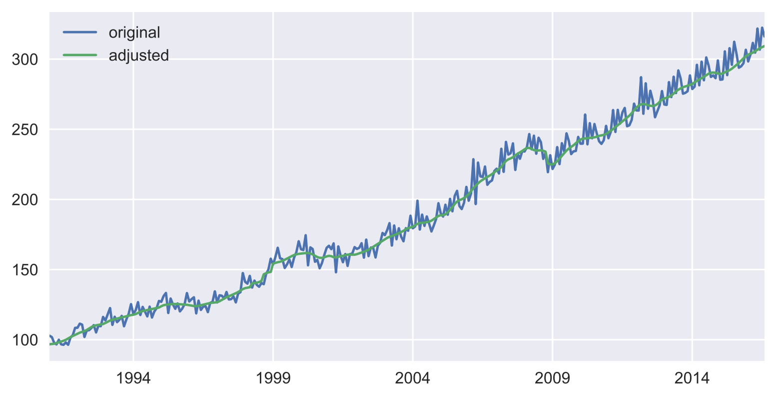
Seasonally adjusted data

- The data we are studying in this lecture, and most data that is used in macro research and in formulating macro policy, is seasonally adjusted.
- That is, in most macro time series, there exists a predictable seasonal component.
- There are various methods for seasonally adjusting data, but the basic idea is to observe historical seasonal patterns and then take out the extra amount that we tend to see on average during a particular season.

Example 1:

Seasonally adjusted IMAE

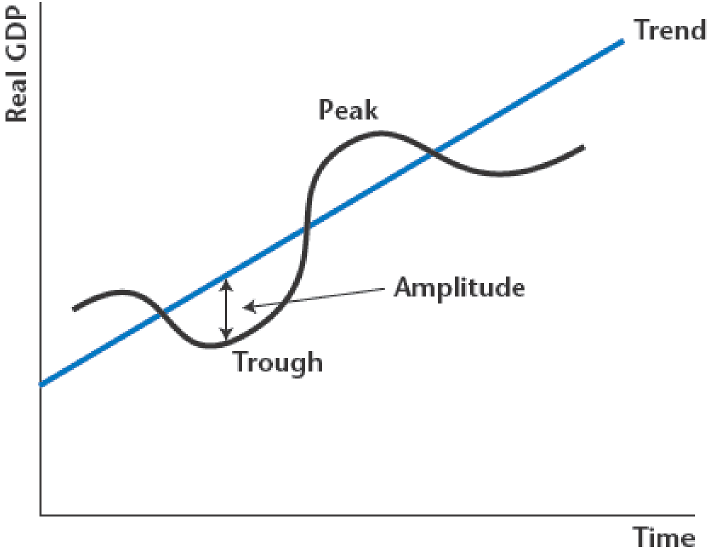
Seasonal adjustment tends to smooth a time series with a seasonal component.



Trend versus cycle

- The primary defining feature of business cycles is that they are fluctuations about trend in real GDP.
- We represent the trend in real GDP with a smooth curve that closely fits actual real GDP, with the trend representing that part of real GDP that can be explained by long-run growth factors.
- What is left over, the deviations from trend, we take to represent business cycle activity.

Idealized business cycle



The need for stationary series

- Many modeling techniques assume that variables are stationary:
  - ARMA
  - DSGE
- To work with non-stationary series, we usually transform (filter) the original data to obtain a stationary series.
- In this lecture, we will analyze the properties of one such transformation: the HP filter.

Separating trend from cycle

- The techniques used to separate trend from cycle are called filters.
- There are plenty of them! For example:

HP	Hodrick-Prescott
FOD	First-Order Differencing
BN	Beveridge-Nelson
UC	Unobservable Components
LT	Linear trend
SEGM	Segmented trend
FREQ	Frequency Domain Masking
MLT	Common deterministic trend
MINDEX	One-dimensional index
COIN	Cointegration

2. The linear trend

Disaggregation of a time series

- We have a sample of T observations on random variable Y_t:

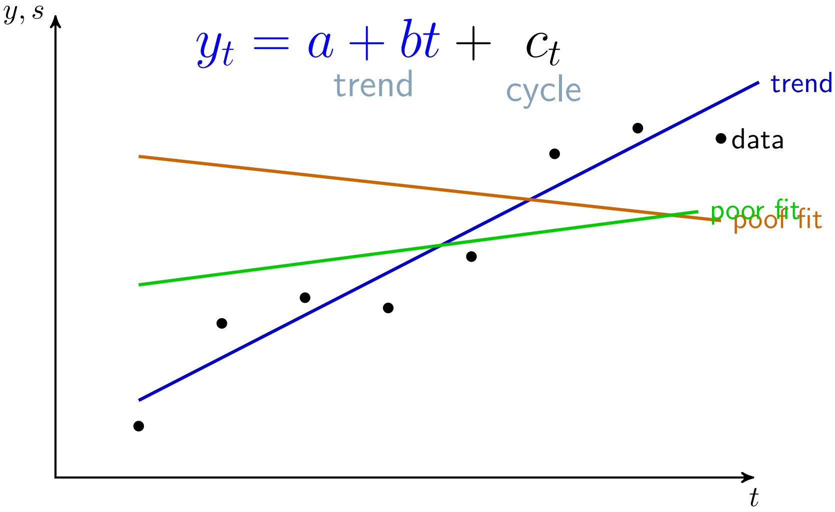
$$\{y_1, y_2, \dots, y_T\}$$

- Y_t has two components: growth (trend) s_t and cycle c_t.

$$y_t = s_t + c_t$$

- We assume that the trend is a straight line, so that $s_t \equiv a + bt$.

Linear trend



How to find the best fit?

- Since data points are not collinear, it is impossible to draw a straight line connecting all of them.
- So we look for a line that passes "close" to all those points.
  - One option is to minimize the sum of all distances between y_t and s_t:
$$\min_{a,b} \sum_{t=1}^{T} |y_t - s_t|$$
  - Another option is to minimize the sum of all squared distances between y_t and s_t:
$$\min_{a,b} \sum_{t=1}^{T} (y_t - s_t)^2$$
- The advantage of the second option is that x^2 is differentiable, while $|x|$ is not.

Finding the linear trend

Formally, the linear trend is defined by $s_t = a^* + b^* t$, where the parameters are such that:

$$a^*, b^* = \underset{a,b}{\operatorname{argmin}} \sum_{t=1}^{T} (y_t - s_t)^2 = \underset{a,b}{\operatorname{argmin}} \sum_{t=1}^{T} (y_t - a - bt)^2$$

First-order conditions are:

$$0 = \sum_{t=1}^{T} (y_t - a - bt) \qquad \Rightarrow \qquad aT + b\Sigma(t) = \Sigma(y)$$

$$0 = \sum_{t=1}^{T} t\,(y_t - a - bt) \qquad \Rightarrow \qquad a\Sigma(t) + b\Sigma(t^2) = \Sigma(ty)$$

Solving for a^* and b^*

$$\begin{bmatrix} T & \Sigma(t) \\ \Sigma(t) & \Sigma(t^2) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \Sigma(y) \\ \Sigma(ty) \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} T & \Sigma(t) \\ \Sigma(t) & \Sigma(t^2) \end{bmatrix}^{-1} \begin{bmatrix} \Sigma(y) \\ \Sigma(ty) \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{T\Sigma(t^2) - \Sigma^2(t)} \begin{bmatrix} \Sigma(t^2) & -\Sigma(t) \\ -\Sigma(t) & T \end{bmatrix} \begin{bmatrix} \Sigma(y) \\ \Sigma(ty) \end{bmatrix}$$

Note that

$$b^* = \frac{T\Sigma(ty) - \Sigma(t)\Sigma(y)}{T\Sigma(t^2) - \Sigma^2(t)} = \frac{\mathrm{Cov}(t, y_t)}{\mathrm{Var}(t)}$$

$$a^*T + b\Sigma(t) = \Sigma(y) \Rightarrow a^* = \bar{y} - b^*\bar{t} = \bar{y} - b^*\tfrac{T+1}{2}$$

A little trick from linear algebra

Let's define these matrices

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix} \qquad S = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_T \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ \vdots & \vdots \\ 1 & T \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{\beta}$$

Rewriting the linear trend problem

$$\beta = \underset{a,b}{\operatorname{argmin}} \sum_{t=1}^{T} (y_t - s_t)^2 = \underset{\beta}{\operatorname{argmin}} (Y - S)'(Y - S)$$

$$= \underset{\beta}{\operatorname{argmin}} (Y - X\beta)'(Y - X\beta)$$

$$= \underset{\beta}{\operatorname{argmin}} \left\{ Y'Y - 2Y'X\beta + \beta'X'X\beta \right\}$$

Side note:

Matrix calculus

Let $x \in \Re^n$, $a \in Re^n$, and A be an $n \times n$ matrix. Then

$$\frac{\partial a'x}{\partial x} = a$$
$$\frac{\partial x'Ax}{\partial x} = (A + A')x$$

and if S is a $n \times n$ symmetric matrix, then

$$\frac{\partial x'Sx}{\partial x} = 2Sx$$

Solving the problem

- Taking the FOC

$$\beta^{OLS} = \underset{\beta}{\operatorname{argmin}} \left\{Y'Y - 2Y'X\beta + \beta'X'X\beta\right\}$$

$$\Rightarrow -2X'Y + 2X'X\beta = 0$$

- Then, the linear trend filter is

$$S^{LT} = X\left(X'X\right)^{-1}X'Y \qquad \text{(trend)}$$

$$C^{LT} \equiv Y - S^{LT} = \left[I - X\left(X'X\right)^{-1}X'\right]Y \qquad \text{(cycle)}$$

Example 2:

The linear trend filter

Assuming that we have $T = 5$ data points $Y = [y_1, y_2, y_3, y_4, y_5]'$ and that $\lambda = 4$, the trend data points $S = [s_1, s_2, s_3, s_4, s_5]'$ are given by :

$$\begin{aligned}
s_1 &= & 0.6y_1 & \quad +0.4y_2 & \quad +0.2y_3 & \quad +0.0y_4 & \quad -0.2y_5 \\
s_2 &= & 0.4y_1 & \quad +0.3y_2 & \quad +0.2y_3 & \quad +0.1y_4 & \quad +0.0y_5 \\
s_3 &= & 0.2y_1 & \quad +0.2y_2 & \quad +0.2y_3 & \quad +0.2y_4 & \quad +0.2y_5 \\
s_4 &= & 0.0y_1 & \quad +0.1y_2 & \quad +0.2y_3 & \quad +0.3y_4 & \quad +0.4y_5 \\
s_5 &= & -0.2y_1 & \quad +0.0y_2 & \quad +0.2y_3 & \quad +0.4y_4 & \quad +0.6y_5
\end{aligned}$$

Notice that each trend data point s_t is just a weighted average of all points in Y. Moreover, some of the weights are negative!

On the other hand, the cycle data points $C = [c_1, c_2, c_3, c_4, c_5]'$ are given by :

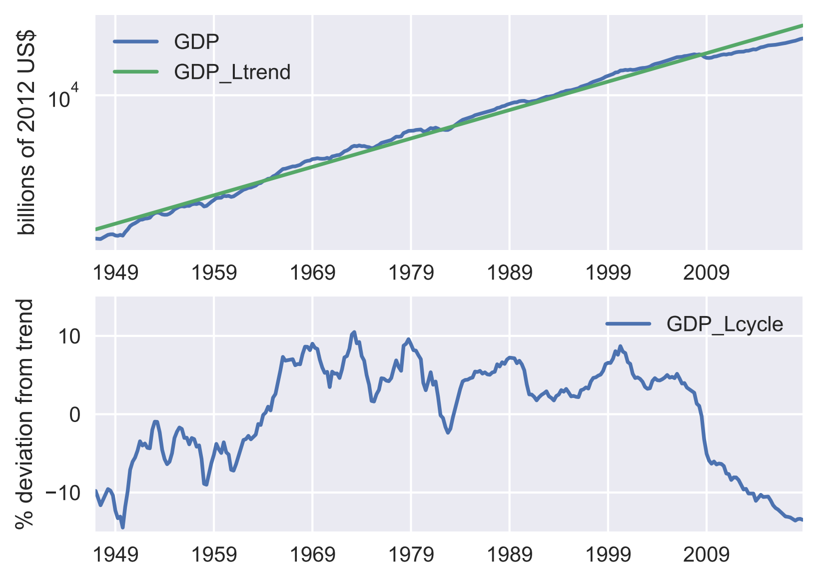
$$
\begin{aligned}
c_1 &= & 0.4y_1 & & -0.4y_2 & & -0.2y_3 & & -0.0y_4 & & +0.2y_5 \\
c_2 &= & -0.4y_1 & & +0.7y_2 & & -0.2y_3 & & -0.1y_4 & & -0.0y_5 \\
c_3 &= & -0.2y_1 & & -0.2y_2 & & +0.8y_3 & & -0.2y_4 & & -0.2y_5 \\
c_4 &= & 0.0y_1 & & -0.1y_2 & & -0.2y_3 & & +0.7y_4 & & -0.4y_5 \\
c_5 &= & 0.2y_1 & & -0.0y_2 & & -0.2y_3 & & -0.4y_4 & & +0.4y_5
\end{aligned}
$$

Again, notice that each cycle data point c_t is just a weighted average of all points in Y.

Example 3:

Filtering with a linear trend

USA real GDP, 2012 dollars



3. The Hodrick-Prescott filter

Disaggregation of a time series

- We have a sample of T observations on random variable Y_t:

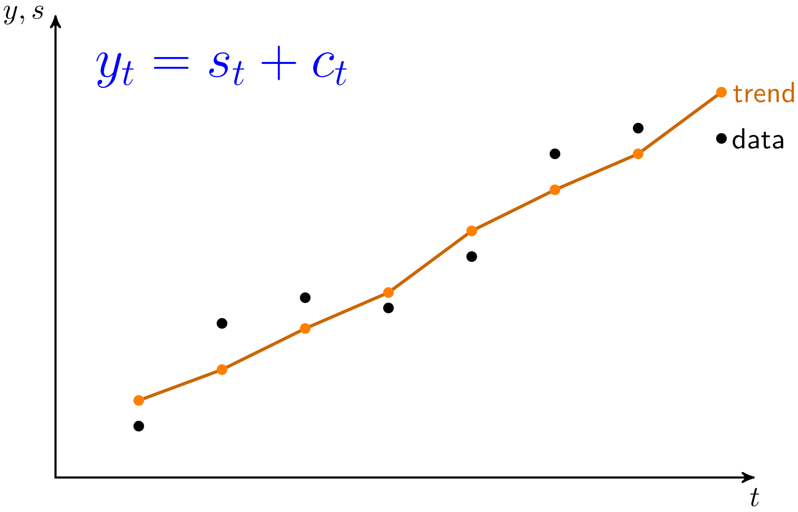
$$\{y_1, y_2, \dots, y_T\}$$

- Y_t has two components: growth (trend) s_t and cycle c_t.

$$y_t = s_t + c_t$$

- We assume that the trend is a *smooth* curve, although not necessarily a straight line.

Data trend



Conflicting objectives

- Starting with y_t, Hodrick and Prescott 1997 "extract" the trend s_t

$$\{s_1, s_2, \dots, s_T\},$$

by balancing two conflicting objectives:

1. the fit to the original series, that is, $y_t - s_t$ must be small.
2. the resulting trend must be smooth, which means that the changes in the slope of the trend $(s_{t+1} - s_t) - (s_t - s_{t-1})$ must be small too.

- The relative importance of these two factors is weighed with a parameter λ.

Formally, the trend is defined by:

$$s_i^{HP} = \underset{s_1,\dots,s_T}{\operatorname{argmin}} \left\{ \sum_{t=1}^{T} (y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1} \left[(s_{t+1} - s_t) - (s_t - s_{t-1}) \right]^2 \right\}$$

$$= \underset{s_1,\dots,s_T}{\operatorname{argmin}} \left\{ \sum_{t=1}^{T} (y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1} (s_{t+1} - 2s_t + s_{t-1})^2 \right\}$$

A little trick from linear algebra

Let's define these matrices

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix} \qquad S = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_T \end{bmatrix}$$

$$A_{T-2\times T} = \begin{bmatrix} 1 & -2 & 1 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & \dots & 0 & 0 & 0 & 0 \\ & & & & \ddots & & & & \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 & -2 & 1 \end{bmatrix}$$

Rewriting the optimization problem

$$
\begin{aligned}
s_i^{HP} &= \underset{s_1,\dots,s_T}{\operatorname{argmin}} \left\{ \sum_{t=1}^{T} (y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1} (s_{t+1} - 2s_t + s_{t-1})^2 \right\} \\
&= \underset{S}{\operatorname{argmin}} \left\{ (Y-S)'(Y-S) + \lambda (AS)'(AS) \right\} \\
&= \underset{S}{\operatorname{argmin}} \left\{ Y'Y - 2Y'S + S'(I + \lambda A'A)S \right\}
\end{aligned}
$$

Solving the problem

- Taking the FOC

$$S^{HP} = \underset{S}{\operatorname{argmin}} \left\{ Y'Y - 2Y'S + S'(I + \lambda A'A)S \right\}$$

$$\Rightarrow -2Y + 2\left(I + \lambda A'A\right) S = 0$$

- Then, the HP filter is

$$S^{HP} = \left(I + \lambda A'A\right)^{-1} Y \qquad \text{(trend)}$$

$$C^{HP} \equiv Y - S^{HP} = \left[I - \left(I + \lambda A'A\right)^{-1}\right] Y \qquad \text{(cycle)}$$

Example 4:

The HP filter trend

Assuming that we have $T = 5$ data points $Y = [y_1, y_2, y_3, y_4, y_5]'$ and that $\lambda = 4$, the trend data points $S = [s_1, s_2, s_3, s_4, s_5]'$ are given by :

$$
\begin{aligned}
s_1 &= & 0.67y_1 & +0.36y_2 & +0.13y_3 & -0.02y_4 & -0.14y_5 \\
s_2 &= & 0.36y_1 & +0.34y_2 & +0.23y_3 & +0.10y_4 & -0.02y_5 \\
s_3 &= & 0.13y_1 & +0.23y_2 & +0.29y_3 & +0.23y_4 & +0.13y_5 \\
s_4 &= & -0.02y_1 & +0.10y_2 & +0.23y_3 & +0.34y_4 & +0.36y_5 \\
s_5 &= & -0.14y_1 & -0.02y_2 & +0.13y_3 & +0.36y_4 & +0.67y_5
\end{aligned}
$$

Notice that each trend data point s_t is just a weighted average of all points in Y. Moreover, some of the weights are negative!

On the other hand, the cycle data points $C = [c_1, c_2, c_3, c_4, c_5]'$ are given by :

$$
\begin{aligned}
c_1 &= \quad 0.33y_1 & -0.36y_2 & \quad -0.13y_3 & +0.02y_4 & \quad +0.14y_5 \\
c_2 &= -0.36y_1 & +0.66y_2 & \quad -0.23y_3 & -0.10y_4 & \quad +0.02y_5 \\
c_3 &= -0.13y_1 & -0.23y_2 & \quad +0.71y_3 & -0.23y_4 & \quad -0.13y_5 \\
c_4 &= \quad 0.02y_1 & -0.10y_2 & \quad -0.23y_3 & +0.66y_4 & \quad -0.36y_5 \\
c_5 &= \quad 0.14y_1 & +0.02y_2 & \quad -0.13y_3 & -0.36y_4 & \quad +0.33y_5
\end{aligned}
$$

Again, notice that each cycle data point c_t is just a weighted average of all points in Y.

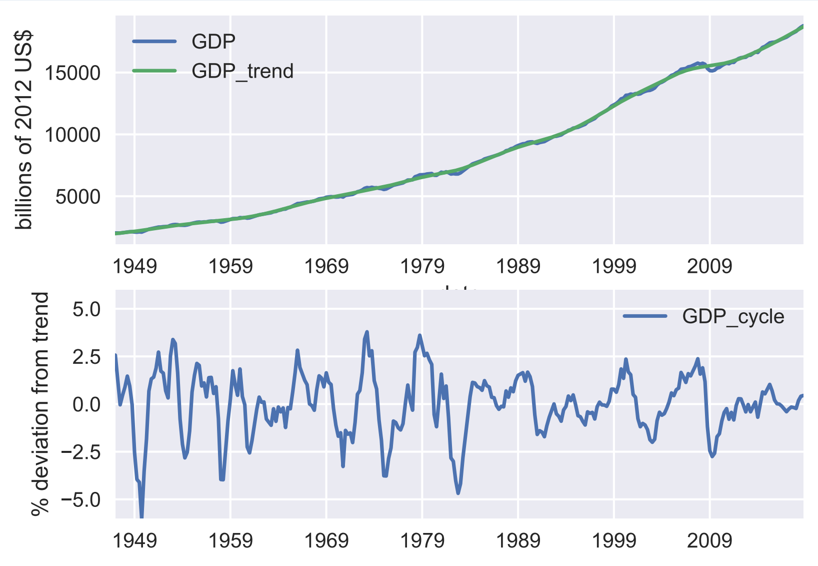
Choosing λ

- The result of filtering is very sensitive to the choice of λ
- As a rule of thumb, λ is chosen depending on frequency of data.
  - Annual $\Rightarrow 100$
  - Quarterly $\Rightarrow 1600$
  - Monthly $\Rightarrow 14400$

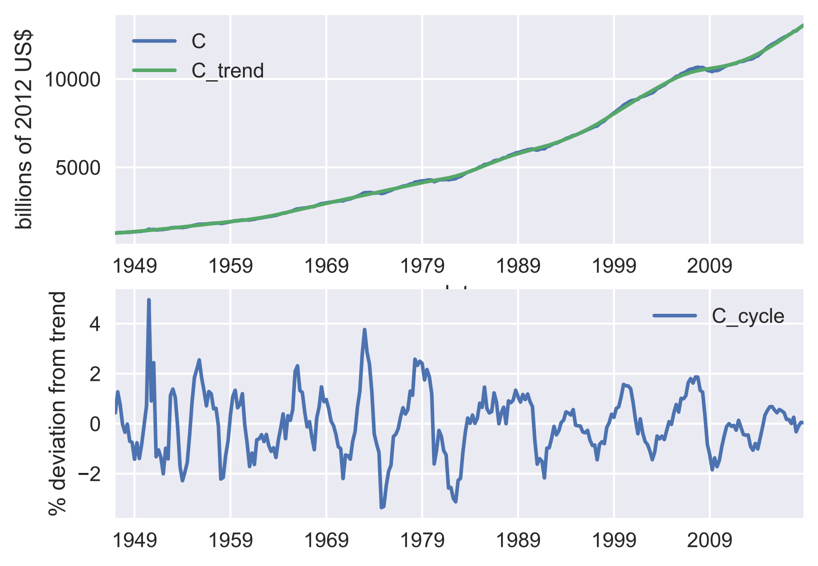
Example 5:

Filtered series when $\lambda = 1600$

USA real GDP, 2012 dollars



USA real consumption, 2012 dollars

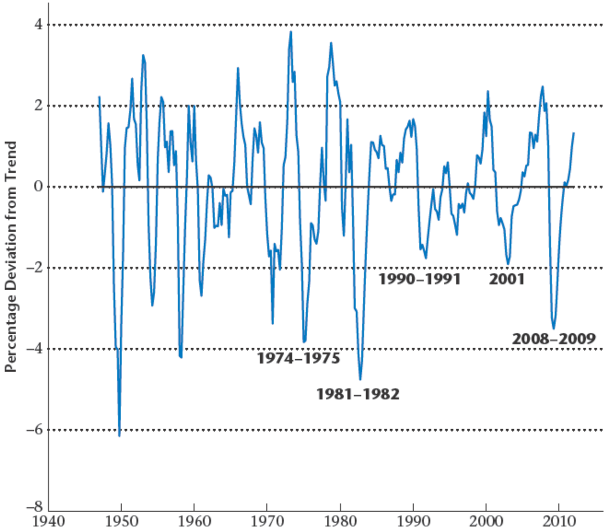


4. Regularities in GDP fluctuations

Forecasting business cycles

- Business cycles are quite irregular: the changes in real GDP are unpredictable; it's very difficult to predict the timing of a business cycle upturn or downturn.
- Business cycles are quite regular, however, in terms of comovements: macroeconomic variables move together in highly predictable ways.

Real GDP cycles from 1947 to 2012



Persistent but irregular

- Real GDP cycles are persistent:
  - when real GDP is above trend, it tends to stay above trend
  - when it is below trend, it tends to stay below trend.
- Real GDP cycles are quite irregular.
  1. The time series of real GDP cycles is quite choppy.
  2. There is no regularity in the amplitude of fluctuations in real GDP about trend. Some of the peaks and troughs represent large deviations from trend, whereas other peaks and troughs represent small deviations from trend.
  3. There is no regularity in the frequency of fluctuations in real GDP about trend. The length of time between peaks and troughs in real GDP varies considerably.

Forecasting implications

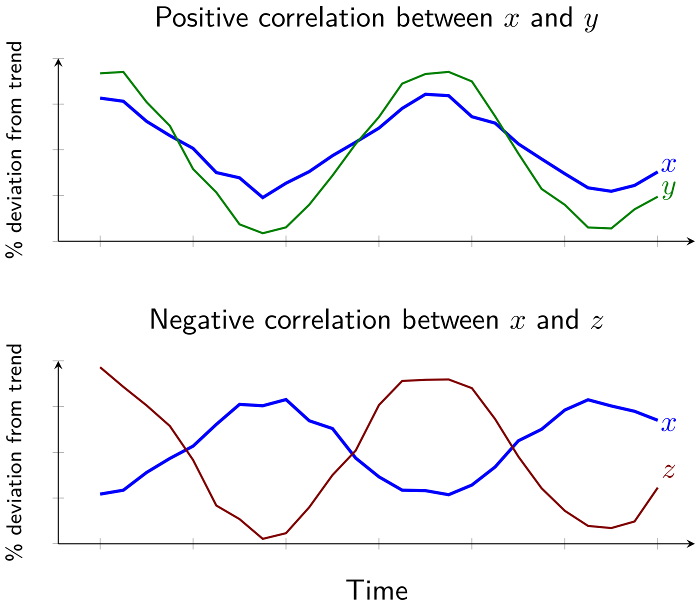
- Because real GDP cycles are persistent, short-term forecasting is relatively easy.
- But because they are irregular longer-term forecasting is difficult:
  - the choppiness of fluctuations in real GDP makes these fluctuations hard to predict
  - the lack of regularity in the amplitude and frequency of fluctuations implies that it is difficult to predict the severity and length of recessions and booms.

5. Comovement

Comovement: looking for (contemporary) correlation

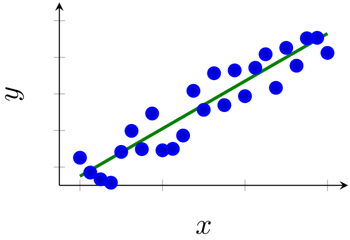
- While real GDP fluctuations are irregular, macro variables fluctuate together in strongly regular patterns.
- We refer to these patterns in fluctuations as comovement.
- Macro variables are measured as time series; for example, real GDP is measured in a series of quarterly observations over time.
- When we examine comovements in macro time series, typically we look at these time series two at a time.
- A good starting point is to plot the data.

Plotting in time domain

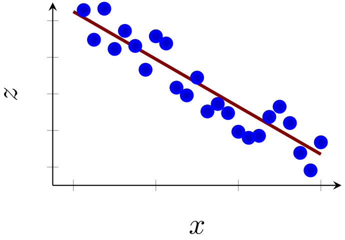


Plotting a scatter plot

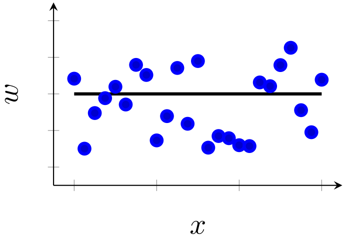
Positive correlation between x and y



Negative correlation between x and z



Zero correlation between x and w



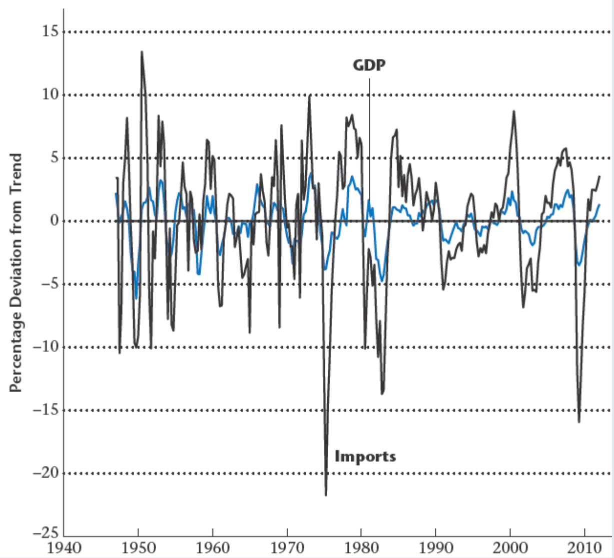
Comovement with real GDP

- Primary interest: how an individual macro variable comoves with real GDP.
- An economic variable is said to be:
  - procyclical if its cycles are positively correlated with the real GDP cycles,
  - countercyclical if its cycles are negatively correlated with the real GDP cycles,
  - acyclical if it is neither procyclical nor countercyclical

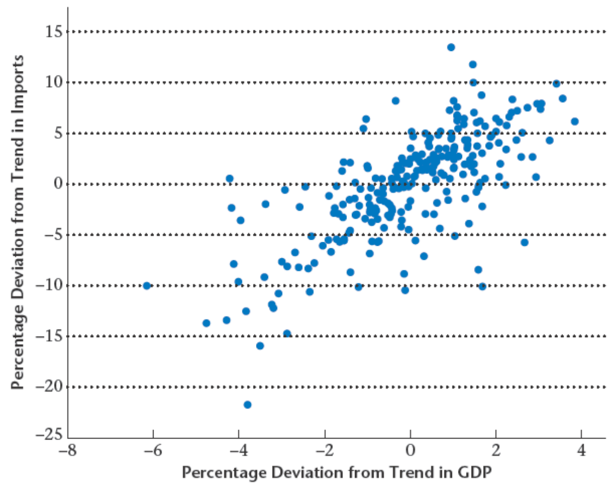
Example 6:

Imports comovement

Imports and GDP are clearly positively correlated, so imports are procyclical.



We again observe the positive correlation between imports and GDP, as a positively sloped straight line would best fit the scatter plot. Again, imports are procyclical.



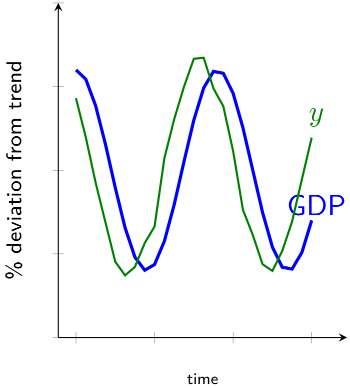
Leading and lagging variables

An important element of comovement is the leading and lagging relationships that exist in macroeconomic data.

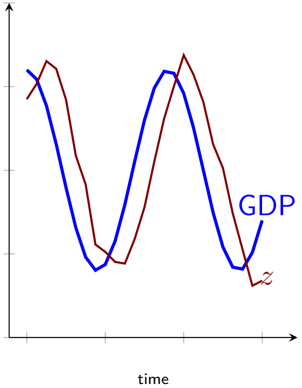
- A leading variable is a macro variable that tends to aid in predicting the future path of real GDP
- If real GDP helps to predict the future path of a particular macroeconomic variable, then that variable is said to be a lagging variable.
- A coincident variable is one which neither leads nor lags real GDP.

Idealized cycles in real GDP and two variables

y is a leading variable



z is a lagging variable



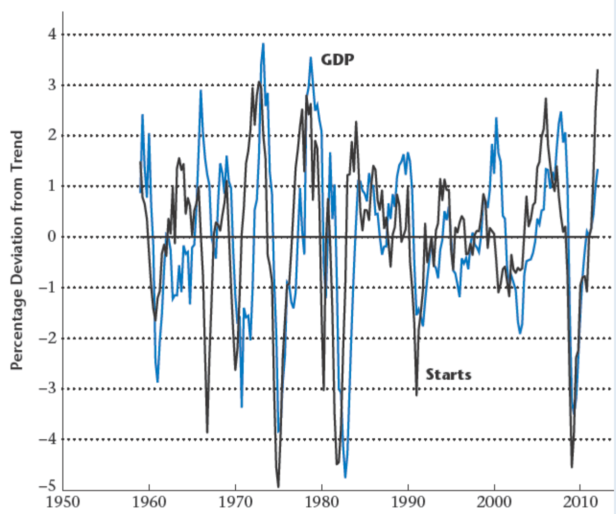
Some leading indicators

- A knowledge of the regularities in leading relationships among economic variables can be very useful in macro forecasting and policymaking.
- Typically, macro variables that efficiently summarize available information about future macro activity are potentially useful in predicting the future path of real GDP.
- For example,
  - the stock market
  - the number of housing starts

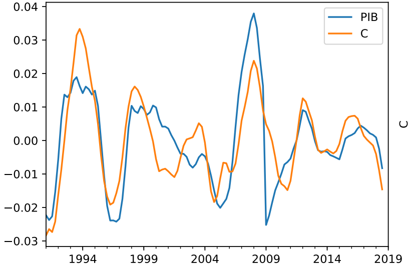
Example 7:

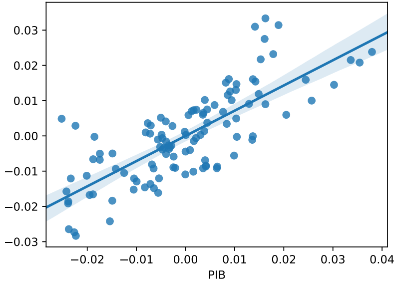
Housing starts as a leading indicator

Percentage deviations in housing starts are divided by 10 so we can see the comovement better. Housing starts clearly lead real GDP (note the timing of turning points in particular).



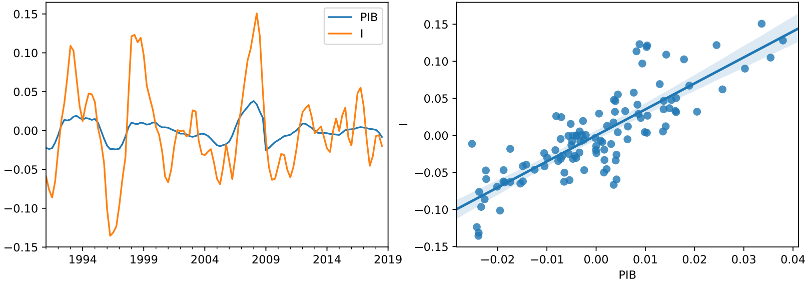
It's procyclical, coincident, and less variable than GDP.



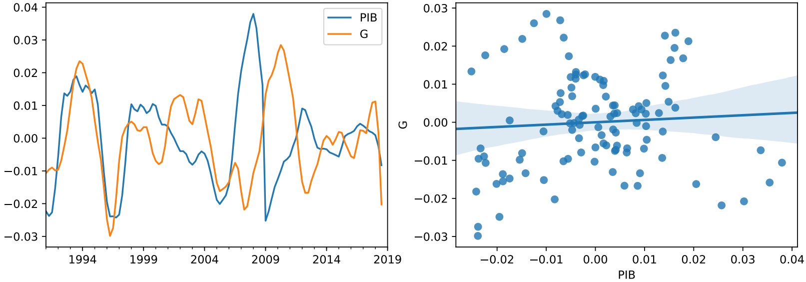


Investment

It's procyclical, coincident, and more variable than GDP.

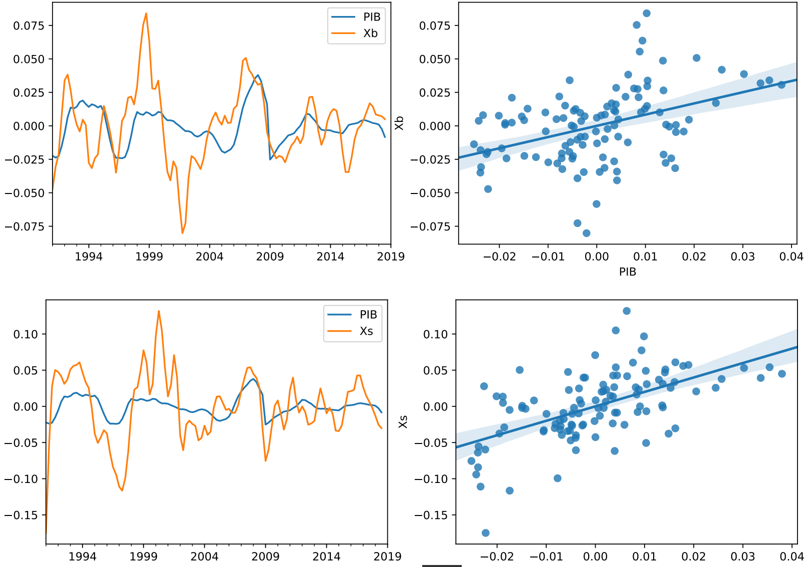


It's procyclical, coincident, and more variable than GDP.



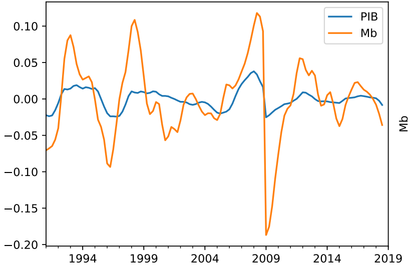
Exports of goods (Xb) and services (Xs)

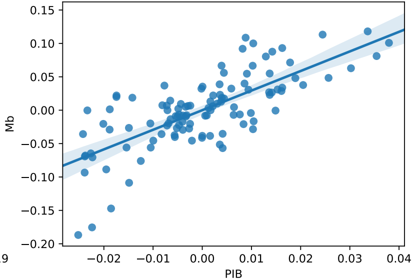
It's procyclical, coincident, and more variable than GDP.

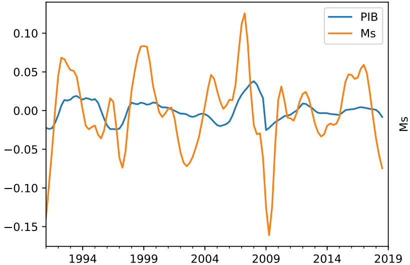


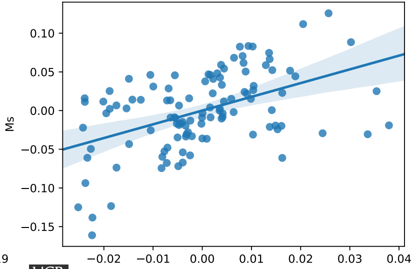
Imports of goods (Mb) and services (Ms)

It's procyclical, coincident, and more variable than GDP.



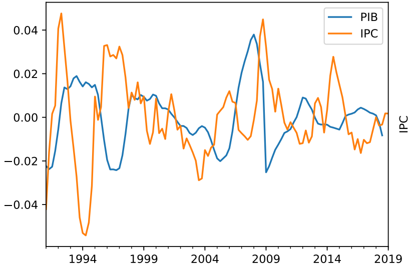


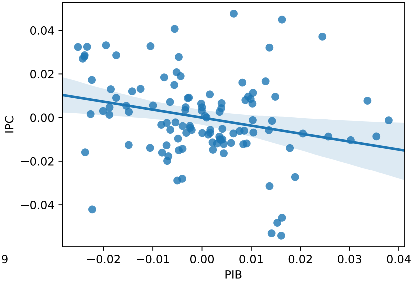




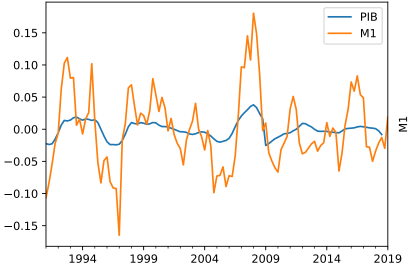
Price level

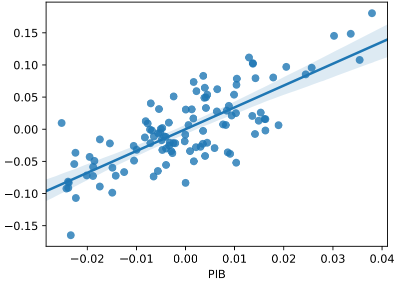
It's countercyclical, coincident, and less variable than real GDP





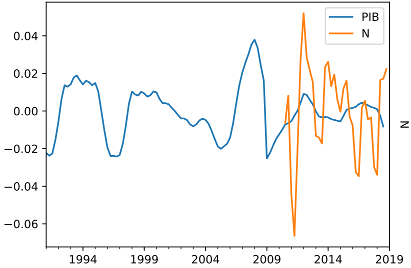
It's procyclical and leading variable, and it is less variable than real GDP.

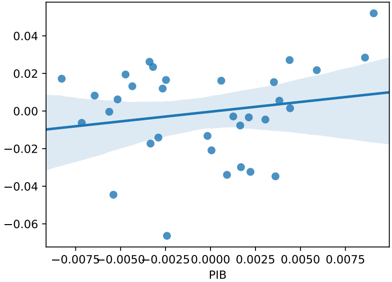




Employment

It's procyclical, it is a lagging variable, and it is less variable than real GDP.





6. Summary of results

Correlation coefficients and variability of cycles

	Correlation Coefficient	Standard Deviation*
PIB	1.00	100.00
C	0.77	93.15
G	0.07	91.76
I	0.82	425.87
Xb	0.41	203.99
Xs	0.57	347.39
Mb	0.74	398.35
Ms	0.48	370.67
IPC	-0.26	140.12
M1	0.78	433.14
N	0.19	181.89

* % of standard deviation of GDP

7. Some warnings against the use of the HP filter

Business cycle "facts" depend on filtering method!

According to Canova 1998, the practice of solely employing the HP1600 filter in compiling business cycle statistics is problematic:

1. The idea that there is a single set of facts which is more or less robust to the exact definition of business cycle is misleading.
2. The empirical characterization of the B.C. obtained with multivariate detrending methods is different from the one obtained with univariate procedures.
3. The practice of building theoretical models whose numerical versions quantitatively match one set of regularities obtained with a particular concept of cyclical fluctuation warrants a careful reconsideration.

U.S. Business Cycle: standard deviations

Filter	GNP	Consumption	Investment	Hours	Real wage	Productivity	Capital
HP1600	1.76	0.49	2.82	1.06	0.70	0.49	0.61
HP4	0.55	0.48	2.70	0.89	0.65	0.69	0.14
FOD	1.03	0.51	2.82	0.91	0.98	0.67	0.63
BN	0.43	0.75	3.80	1.64	2.18	1.14	2.64
UC	0.38	0.34	6.72	4.14	2.24	4.09	1.22
LT	4.03	0.69	2.16	0.69	1.71	1.00	1.56
SEGM	2.65	0.52	3.09	1.01	1.10	0.54	0.97
FREQ1	1.78	0.46	3.10	1.20	1.07	0.66	1.41
FREQ2	1.14	0.44	3.00	1.16	1.11	0.69	1.26
MLT	6.01	0.67	2.36	0.46	1.21	1.00	1.05
MINDEX	3.47	0.98	2.65	1.14	1.27	0.72	1.85
COIN	4.15	0.71	3.96	0.75	1.68	1.09	1.30

(absolute for GNP, all others relative to GNP)

HP has bad statistical properties

Hamilton 2017: Why You Should Never Use the HP Filter?

1. The Hodrick-Prescott (HP) filter introduces spurious dynamic relations that have no basis in the underlying data-generating process.
2. Filtered values at the end of the sample are very different from those in the middle and are also characterized by spurious dynamics.
3. A statistical formalization of the problem typically produces values for the smoothing parameter vastly at odds with common practice.
4. There is a better alternative. A regression of the variable at date t on the four most recent values as of date $t - h$ achieves all the objectives sought by users of the HP filter with none of its drawbacks.

HP can induce wrong conclusions about series comovement

Cogley and Nason (1995) analyze the spectral properties of the HP filter

- When measuring the business component of a time series, is it a good idea to use the Hodrick-Prescott filter?

  Yes! if original series is Trend Stationary
  No! if original series is Difference-Stationary

- Implications for DSGE models
  - When applied to integrated processes, the HP filter can generate business cycle periodicity and comovement even if none are present in the original data.
  - Standard real business cycle models do not generate business cycle dynamics in pre-filtered data.
  - The business cycles observed in HP filtered data are due to the filter.

8. The Baxter-King filter

The Baxter-King filter

- We can imagine that a time series $\{y_t\}$ is the result of adding up an infinite number of periodic functions, where a given function repeats itself every p periods (that is, it has a frequency $\omega = \frac{2\pi}{p}$).
- In the Baxter and King 1999 filter, we start by explicitly defining the business cycles as the sum of those periodic functions where $p \in [p_L, p_H]$.
- This corresponds to periodic functions with frequencies $\omega \in [\omega_L, \omega_H] = \left[\frac{2\pi}{p_H}, \frac{2\pi}{p_L}\right]$.
- Following Burns-Mitchell definition of the business cycle, we usually set $p \in [6, 32]$ quarters.

- The ideal filter would then measure the cycle c_t by the symmetric moving average:

$$c_t = \sum_{k=-\infty}^{\infty} b_k y_{t-k}$$

where the weights are given by

$$b_0 = \frac{\omega_H - \omega_L}{\pi},$$

$$b_k = \frac{\sin(k\omega_H) - \sin(k\omega_L)}{k\pi} \quad \text{for } k \neq 0$$

- Notice that we cannot use this ideal filter, because we do not have infinite data!

- In practice, we truncate the ideal filter to using only K lags:

$$c_t = \sum_{k=-K}^{K} b_k^* y_{t-k}$$

where the adjusted weights b_k^* are given by

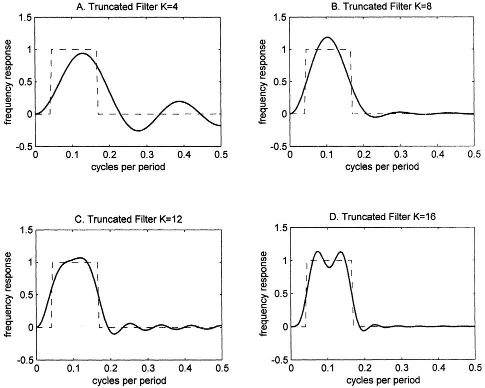
$$b_k^* = b_k - \tfrac{1}{2K+1} \sum_{h=-K}^{K} b_h$$

- The last term (adjusment) is made so that weights add up to zero, which is necessary to cancel the trend in the data.

The Baxter-King filter (cont'n)

- Notice the trade-off in setting the value of K:
  - if K is too small, we get far from the ideal filter
  - if K is too high, we "lose" bigger tails in the data.

Figure 2.—Constrained Approximate High-Pass Filters



Example 8:

The Baxter-King filter

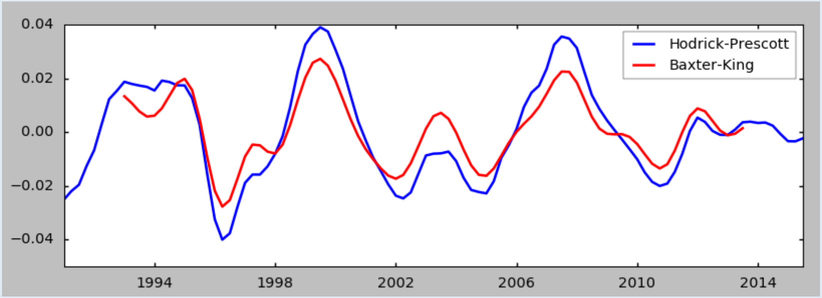
Suppose we set $p_L = 6, p_H = 32, K = 3$. In that case the cycle is given by:

$$c_t = -0.164y_{t-3} - 0.028y_{t-2} + 0.109y_{t-1} + 0.166y_t + \dots \\ + 0.109y_{t+1} - 0.028y_{t+2} - 0.164y_{t+3}$$

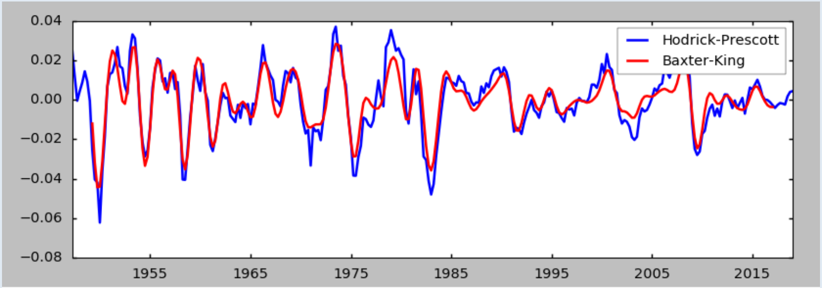
Example 9:

Baxter-King vs the Hodrick Prescott

Business cycle in Costa Rica, Hodrick Prescott ($\lambda = 1600$) versus Baxter-King ($K = 8, p \in [6, 32]$)



Business cycle in the United States, Hodrick Prescott ($\lambda = 1600$) versus Baxter-King ($K = 8, p \in [6, 32]$)



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