



# Lecture 1

## Introduction to Computational Economics

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Universidad de Costa Rica

SP6534 - Economía Computacional

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# Introduction

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- This is one of the most important equations in economics:

$$V(s) = \max_x \{f(s, x) + \delta V(g(s, x))\}.$$

- It is called the Bellman equation.
- It describes the behavior of a rational economic agent who weighs the future consequences of the actions he takes today.
- It is a functional equation.
- You do not know how to solve functional equations.
- But you will by the conclusion of this course.

## Some Apparently Simple Questions

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# Inverse Demand

- Consider the constant elasticity demand function

$$q = p^{-0.5}.$$

- What price  $p^*$  clears the market of a quantity  $q = 2$ ?
- Easy. Raise both sides of the equation by a power of  $-2$  and flip to derive the *inverse demand function*

$$p = q^{-2}.$$

- Then compute the market clearing price

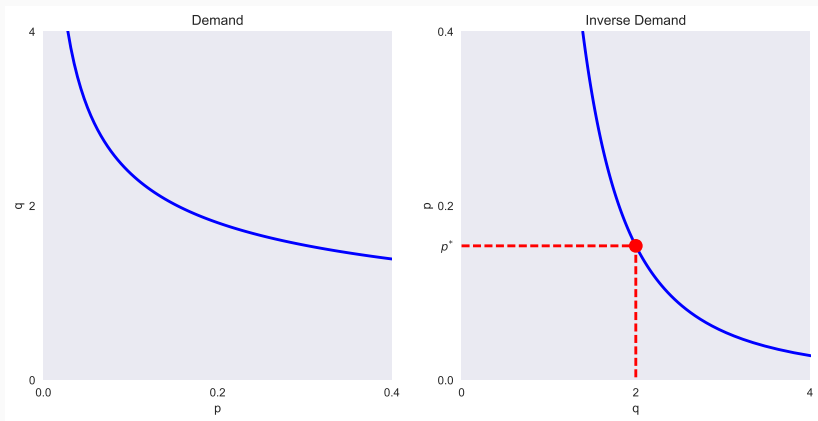
$$p^* = 2^{-2} = 1/4.$$

- Now consider the demand function

$$q = 0.5p^{-0.2} + 0.5p^{-0.5}$$

with two additive terms, say, domestic and export demand.

- Clearly, the demand function is strictly decreasing and continuously differentiable.
- By the Implicit Function Theorem, the inverse demand function is well-defined, strictly decreasing, and differentiable.
- But what is the inverse demand function and what price  $p^*$  clears the market of a quantity  $q = 2$ ?



**Figure 1:** Demand function  $q = 0.5p^{-0.2} + 0.5p^{-0.5}$  and its inverse



# Price Support Program

- Consider an agricultural market with two periods: planting and harvest.
- Farmers plant acreage  $a$  based on the price  $p$  they expect to receive at harvest:

$$a = 0.5 + 0.5Ep.$$

- Market clearing price at harvest is a function of output  $q$ :

$$p = 1.5 - 0.5q.$$

- Output  $q$  at harvest depends on acreage planted and an exogenous random per-acre yield  $\tilde{y}$ :

$$q = a\tilde{y}.$$

- Yield unknown at planting, but its expectation is 1.

- To solve, substitute the third expression into the second

$$p = 1.5 - 0.5a\tilde{y}.$$

- Take expectations of both sides

$$Ep = 1.5 - 0.5a.$$

- Substitute into the first expression

$$a = 0.5 + 0.5(1.5 - 0.5a) = 1.25 - 0.25a.$$

- Conclude that

$$a = 1 \quad \text{and} \quad Ep = 1.$$

- President Carlos Alvarado proposes a price support program.
- If the market price  $p$  falls below a target of 1, the government will pay each farmer  $1 - p$  per unit of output.
- Thus, the effective farm price is  $\max(p, 1)$ .
- Farmers continue to plant acreage based on the price they expect to receive at harvest:

$$a = 0.5 + 0.5E \max(p, 1).$$

- The model is now

$$a = 0.5 + 0.5E \max(p, 1)$$

$$p = 1.5 - 0.5q$$

$$q = a\tilde{y}$$

- So, after some substitutions,

$$a = 0.5 + 0.5E \max(1.5 - 0.5a\tilde{y}, 1).$$

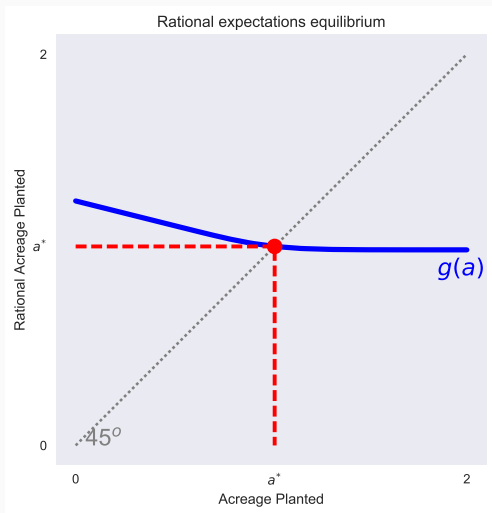
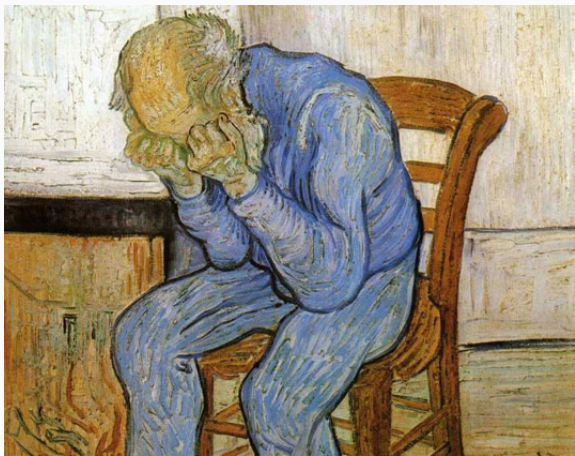


Figure 2: Rational expectations equilibrium

- The phone rings.
- It is President Alvarado.
- He asks: How will program affect prices paid by consumers, prices received by farmers, and government expenditures?
- He wants an answer from you by tomorrow morning.
- What are you going to tell him?



# An Alternative Analytic Framework

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# An Alternative Analytic Framework

- The two models above cannot be solved using standard methods of Algebra and Calculus:
  - First model poses a quintic polynomial rootfinding problem.
  - Second model poses nonlinear fixed-point problem.
- Both models lack closed-form solution.
- They can, however, be easily solved on a computer using *numerical methods*.

# Inverse Demand

- Given the demand function

$$q = 0.5p^{-0.2} + 0.5p^{-0.5},$$

the price  $p^*$  that clears the market of a quantity  $q = 2$  is a root of the function

$$f(p) = 0.5p^{-0.2} + 0.5p^{-0.5} - 2,$$

that is,  $f(p^*) = 0$ .

- The root of  $f$  can be computed using *Newton's Method*: guess an initial value  $p_0$  and successively form the iterates

$$p_{k+1} = p_k - \frac{f(p_k)}{f'(p_k)}$$

until they converge.

- Open the CompEcon notebook [demintro01](#), which computes the market clearing price using Newton's method:

```
p = 0.2
for it in range(50):
    f = 0.50*p** -0.2 + 0.50*p** -0.5 - 2
    d = -0.01*p** -1.2 - 0.25*p** -1.5
    s = -f / d
    p += s
    if abs(s) < 1e-10:
        break
```

- Execute the script. After 14 iterations,  $p$  converges to 0.1542.

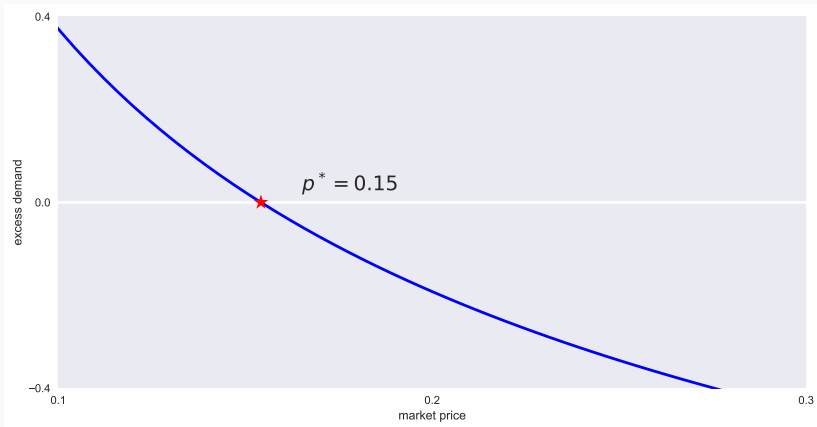


Figure 3: Excess demand function  $q = 0.5p^{-0.2} + 0.5p^{-0.5} - 2$  and its root

# Price Support Program

- Under the price support program, the equilibrium acreage  $a$  is a “fixed-point” of the function

$$g(a) = 0.5 + 0.5E \max(1.5 - 0.5a\tilde{y}, 1),$$

that is,  $a = g(a)$ .

- The fixed-point of  $g$  may be computed using *function iteration*: guess an initial value  $a_0$  and successively form the iterates

$$a_{k+1} = g(a_k)$$

until they converge.

Open CompEcon notebook [demintro02](#), which computes the equilibrium acreage using function iteration, assuming yield is lognormal with mean 1 and volatility 0.2:

```
from compecon import qnwlogn
import numpy as np

sigma2 = 0.2**2
y, w = qnwlogn(25, -0.5*sigma2, sigma2)

def acreage(aa, pp):
    return 0.5 + 0.5*w @ np.maximum(1.5-0.5*aa*y, pp)

a, ptarg = 1, 1
for it in range(50):
    aold, a = a, acreage(a, ptarg)
    if np.linalg.norm(a - aold) < 1.e-8:
        break
```

Execute the script. After 9 iterations,  $a$  converges to 1.0174.

- Now let us answer President Alvarado's questions.
- Run `demintro02`, which computes the means and standard deviations of quantity produced, market price, farm price, farm revenue, and government expenditure.
- You should get

Variable	Expect	Std Dev
Market Price	0.9913	0.1028
Farm Price	1.0348	0.0506
Farm Revenue	1.0447	0.1773
Government Expenditures	0.0573	0.1038

- How do these values change as the target price changes?

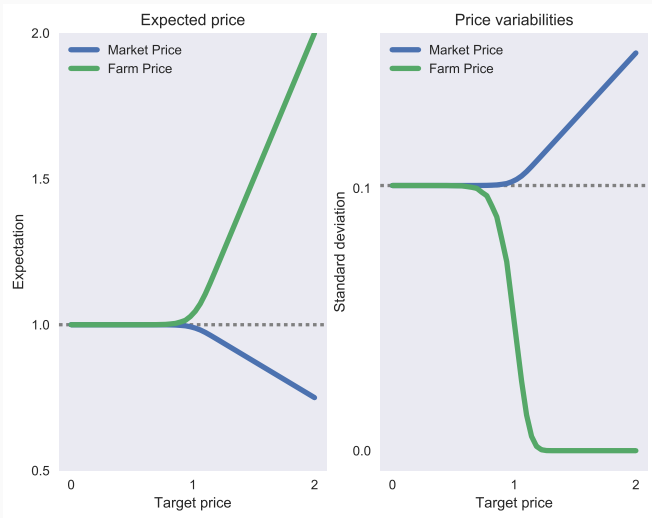


Figure 4: Price expectation and volatility



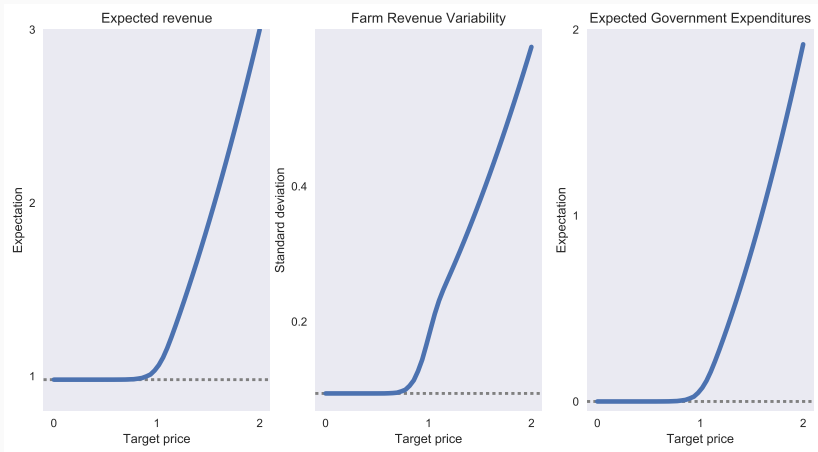


Figure 5: Farm revenue and government expenditure

# About the Course

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# Why Numerical Methods?

Many interesting economic models lack closed-form solution and cannot be solved analytically using Algebra and Calculus, including

- Large-scale or highly nonlinear models.
- Models with binding structural constraints.
- Dynamic models, which often give rise to functional equations.

Many of these models, however, may be solved and analyzed using numerical methods.

- Biological, physical, and engineering sciences have readily embraced numerical methods.
- Economists, however, have resisted.
- Widely-held views are that numerical solutions are less elegant or less general than closed-form solutions.
- These propositions, however, are debatable.

- When essential features of an economic system cannot be faithfully captured in an algebraically soluble model, a choice must be made.
- Either ignore the essential features to render an algebraically soluble model, or use an alternative analytic framework.
- The former choice, unfortunately, is made too often.

- Numerical methods, however, are rapidly gaining acceptance among economists.
- With the growth of computational power, the relative benefits of finding exact closed-form solutions has diminished.
- In many economic applications, simple, approximate solutions are often more useful than complex exact solutions.
- Moreover, poor data quality and limited understanding of underlying structural relationships may not justify exactness.

# Course Objective

To train you in the use numerical methods to formulate, solve, and analyze economic and financial models, with emphasis on dynamic models.

Upon completion of course, students should possess skills to

- formulate, solve, and analyze economic models using advanced mathematical, statistical, and numerical methods
- master material in scholarly Economics literature
- master material in advanced courses in Economics
- conduct rigorous research in any field of Economics



# Course Structure

- Dynamic economic models will be stressed.
- Course based on Miranda and Fackler (2002) textbook.
- Examples drawn from many areas of economics and finance.
- Agent-based, computable general equilibrium, and network models not covered.

- Course is hands-on: learn to use Python, CompEcon Toolbox.
- Python applications integrated into lectures.
- Install Python on your personal computer, bring it to class.

# Prerequisites

- Calculus
- Linear Algebra
- Mathematical Statistics
- Intermediate Microeconomic Theory

- Linear and Nonlinear Equations
- Finite-Dimensional Optimization
- Numerical Integration and Differentiation
- Numerical Functional Equation Methods
- Ordinary Differential Equations
- Discrete Time Dynamic Programming

## References

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Miranda, Mario J. and Paul L. Fackler (2002). *Applied Computational Economics and Finance*. MIT Press. ISBN: 0-262-13420-9.