Lecture 3

Expectations: The Basic Tools

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Universidad de Costa Rica
EC3201 - Teoría Macroeconómica 2
1. Nominal versus Real Interest Rates

2. Nominal and Real Interest Rates and the IS-LM Model

3. Expected Present Discounted Values
• In Lecture 1 about history of Macro, we emphasized that since the 1970s economist pay special attention to the role of expectations.

• In this lecture se introduce the basic tools.

• In the next few lectures, we analyze how expectations affect:
  • financial markets (Lecture 4).
  • consumption and investment (Lecture 5)
  • output and macroeconomic policy (Lecture 6)

• This topic is very important, because many economic decisions depend not only on what is happening now, but also on what people expect about the future.
Example 1:
Decisions that depend on expectations about the future
• A consumer: “Can I afford a loan to pay for a new car?”

• A company manager: “Sales have increased in past few years. Will they keep this trend in the near future? Should I buy more machines?”

• A pension fund manager: “The stock market is plummeting. Is it going to rebound soon? Should I divest and look for safe assets?”
Nominal versus Real Interest Rates
Nominal versus real interest rates

- Interest rates expressed in terms of dollars (or, more generally, in units of the national currency) are called **nominal interest rates**.
- Interest rates expressed in terms of a basket of goods are called **real interest rates**.

In what follows:

\[
i_t \quad \text{nominal interest rate for year } t.
\]

\[
r_t \quad \text{real interest rate for year } t.
\]

\[
1 + i_t \quad \text{lending one dollar this year yields } (1 + i_t) \text{ dollars next year. Alternatively, borrowing one dollar this year implies paying back } (1 + i_t) \text{ dollars next year.}
\]

\[
P_t \quad \text{price of basket of goods this year.}
\]

\[
P_{t+1}^e \quad \text{expected price next year.}
\]
How nominal and real interest rates are related

Chapter 6
Financial Markets II: The Extended IS-LM Model

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If it is the one-year nominal interest rate—the interest rate in terms of dollars—and if you borrow $P_t$ dollars, you will have to repay $(1 + i_t)P_t$ dollars next year. This is represented by the arrow from left to right at the bottom of Figure 6-1.

What you care about, however, is not dollars, but pounds of bread. Thus, the last step involves converting dollars back to pounds of bread next year. Let $P_{t+1}$ be the price of bread you expect to pay next year. (The superscript $e$ indicates that this is an expectation; you do not know yet what the price of bread will be next year.) How much you expect to repay next year, in terms of pounds of bread, is therefore equal to $1 + i_tP_t$ (the number of dollars you have to repay next year) divided by $P_{t+1}$ (the price of bread in terms of dollars expected for next year), so $(1 + i_tP_t) > P_{t+1}$. This is represented by the arrow pointing up in the lower right of Figure 6-1.

Putting together what you see in both the top part and the bottom part of Figure 6-1, it follows that the one-year real interest rate, $r_t$, is given by:

$$1 + r_t = \frac{(1 + i_t)P_t}{P_{t+1}}$$

This relation looks intimidating. Two simple manipulations make it look much friendlier:

1. Denote expected inflation between $t$ and $t+1$ by $p_t$. Given that there is only one good—bread—the expected rate of inflation equals the expected change in the dollar price of bread between this year and next year, divided by the dollar price of bread this year:

$$p_t = \frac{P_{t+1} - P_t}{P_t}$$

2. Using equation (6.2), rewrite $P_t > P_{t+1}$ in equation (6.1) as $1 > 1 + p_t$. Replace $i_t$ in equation (6.1) to get:

$$1 + r_t = 1 + p_t$$

Add 1 to both sides in equation (6.2):

$$1 + p_t = 1 + \frac{P_{t+1} - P_t}{P_t}$$

Reorganize:

$$1 + p_t = \frac{P_{t+1}}{P_t}$$

Take the inverse on both sides:

$$1 + r_t = \frac{P_{t+1}}{P_t}$$

Replace in equation (6.1) and you get equation (6.3).
A useful approximation

• Given \( 1 + r_t = (1 + i_t) \frac{P_t}{P_{t+1}} \), and knowing that

\[
\frac{P_{t+1}}{P_t} = 1 + \pi_{t+1}^e, \text{ then}
\]

\[
1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}^e}
\]

• If the nominal interest rate and the expected rate of inflation are not too large, a simpler expression is:

\[
r_t \approx i_t - \pi_{t+1}^e
\]

• The real interest rate is (approximately) equal to the nominal interest rate minus the expected rate of inflation.
Real vs nominal interest rate: some implications

\[ r_t \approx i_t - \pi^e_{t+1} \]

Here are some of the implications of the relation above:

• If \( \pi^e_{t+1} = 0 \), then \( i_t = r_t \)
• If \( \pi^e_{t+1} > 0 \), then \( i_t > r_t \)
• If \( \ddot{i}_t \) is constant, then \( \uparrow \pi^e_{t+1} \) implies \( \downarrow r_t \)
Example 2:
Nominal and real interest rates in Costa Rica
Although the nominal interest rate has declined during the last decade, the real interest rate was actually higher in 2016 than in 2006.
Which interest rate should enter the IS relation?

• Consumers and investors base their decisions on the real interest rate.
• This has a straightforward implication for monetary policy.
• Although the central bank chooses the nominal rate, it cares about the real interest rate because this is the rate that affects spending decisions.
• To set the real interest rate it wants, it thus has to take into account expected inflation.
• If, for example, it wants to set the real interest rate equal to $r$, it must choose the nominal rate $i$ so that, given expected inflation, $\pi^e$, the real interest rate, $r = i - \pi^e$, is at the level it desires.

• For example, if it wants the real interest rate to be 4%, and expected inflation is 2%, it will set the nominal interest rate, $i$, at 6%.

• So, we can think of the central bank as choosing the real interest rate.
The Zero Lower Bound and Deflation

- The zero lower bound implies that $i \geq 0$; otherwise people would not want to hold bonds.
- This implies that the $r \geq -\pi^e$.
- So long as $\pi^e > 0$, this allows for negative real interest rates.
- But if $\pi^e$ turns negative, if people anticipate deflation, then the lower bound on $r$ is positive and can turn out to be high.
- This may not be low enough to increase the demand for goods by much, and the economy may remain in recession.
- The zero lower bound turned out to be a serious concern during the 2008 crisis.
Until now, we assumed there was only one type of bond. Bonds however differ in a number of ways. For example, maturity, risk.

Some bonds are risky, with a non-negligible probability that the borrower will not be able or willing to repay.

To compensate for the risk, bond holders require a risk premium.
What determines this risk premium?

1: The probability of default itself.

- The higher this probability, the higher the interest rate investors will ask for.
- More formally, let \( i \) be the nominal interest rate on a riskless bond, and \( i + x \) be the nominal interest rate on a risky bond, which is a bond which has probability, \( p \), of defaulting. Call \( x \) the risk premium.
- Then, to get the same expected return on the risky bonds as on the riskless bond, the following relation must hold:

\[
1 + i = (1 - p)(1 + i + x) + (p)(0) \quad \Rightarrow \quad x = \frac{(1 + i)p}{1 - p}
\]

- So for example, if \( i = 4\% \), and \( p = 2\% \), then the risk premium required to give the same expected rate of return as on the riskless bond is equal to 2.1\%. 
What determines this risk premium?

2: The degree of risk aversion of the bond holders

- Even if the expected return on the risky bond was the same as on a riskless bond, the risk itself will make them reluctant to hold the risky bond.
- Thus, they will ask for an even higher premium to compensate for the risk.
- How much more will depend on their degree of risk aversion.
- And, if they become more risk averse, the risk premium will go up even if the probability of default itself has not changed.
In September 2008, the financial crisis led to a sharp increase in the rates at which firms could borrow.

Figure 6-3
Yields on 10-Year U.S. Government Treasury, AAA, and BBB Corporate Bonds, since 2000

Note three things about the figure. First, the rate on even the most highly rated (AAA) corporate bonds is higher than the rate on U.S. government bonds, by a premium of about 2% on average. The U.S. government can borrow at cheaper rates than U.S. corporations. Second, the rate on lower rated (BBB) corporate bonds is higher than the rate on the most highly rated bonds by a premium often exceeding 5%. Third, note what happened during 2008 and 2009 as the financial crisis developed. Although the rate on government bonds decreased, reflecting the decision of the Fed to decrease the policy rate, the interest rate on lower-rated bonds increased sharply, reaching 10% at the height of the crisis. Put another way, despite the fact that the Fed was lowering the policy rate down to zero, the rate at which lower rated firms could borrow became much higher, making it extremely unattractive for these firms to invest. In terms of the IS-LM model, this shows why we have to relax our assumption that it is the policy rate that enters the IS relation. The rate at which many borrowers can borrow may be much higher than the policy rate.
Nominal and Real Interest Rates and the IS-LM Model
Nominal and real interest rates and the IS–LM model

- When deciding how much investment to undertake, firms care about real interest rates. Then, the IS relation must read:

\[ Y = C(Y - T) + I(Y, i + x - \pi^e) + G \]

- The central bank still controls the nominal interest rate:

\[ i = \bar{i} \]

- The real interest rate is:

\[ r = i - \pi^e \]
• Although the central bank formally chooses the nominal interest rate, it can choose it in such a way as to achieve the real interest rate it wants (this ignores the issue of the zero lower bound).

• Thus, we can think of the central banks as choosing the real policy rate directly and rewrite the two equations as:

\[ Y = C(Y - T) + I(Y, r + x) + G \]  \hspace{1cm} (IS)

\[ r = \bar{r} \]  \hspace{1cm} (LM)
An increase in $x$ leads to a shift of the IS curve to the left and a decrease in equilibrium output.
If sufficiently large, a decrease in the policy rate can in principle offset the increase in the risk premium. The zero lower bound may however put a limit on the decrease in the real policy rate.
The financial crisis led to a shift of the IS to the left. Financial and fiscal policies led to some shift back of the IS to the right. Monetary policy led to a shift of the LM curve down. Policies were not enough however to avoid a major recession.
From monetary policy to output

Note an immediate implication of these three relations:

- The interest rate directly affected by monetary policy is the nominal interest rate.
- The interest rate that affects spending and output is the real interest rate.
- So, the effects of monetary policy on output depend on how movements in the nominal interest rate translate into movements in the real interest rate.
Expected Present Discounted Values
The value of money over time

The expected present discounted value of a sequence of future payments is the value today of this expected sequence of payments.

(a) One dollar this year is worth $1 + i_t$ dollars next year.

(b) If you lend/borrow $\frac{1}{1+i_t}$ dollars this year, you will receive/repay $\frac{1}{1+i_t}(1 + i_t) = 1$ dollar next year.

(c) One dollar is worth $(1 + i_t)(1 + i_{t+1})$ dollars two years from now.

(d) The present discounted value of a dollar two years from today is equal to $\frac{1}{(1+i_t)(1+i_{t+1})}$. 

<table>
<thead>
<tr>
<th>This year</th>
<th>Next year</th>
<th>2 years from now</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$(1 + i_t)$</td>
<td>$1$</td>
</tr>
<tr>
<td>$\frac{1}{1+i_t}$</td>
<td>$1$</td>
<td>$(1 + i_t)(1 + i_{t+1})$</td>
</tr>
<tr>
<td>$\frac{1}{(1+i_t)(1+i_{t+1})}$</td>
<td></td>
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The word “discounted” comes from the fact that the value next year is discounted, with \((1 + i_t)\) being the discount factor. The 1-year nominal interest rate, \(i_t\), is sometimes called the discount rate.
Computing expected present discounted values

• The present discounted value of a sequence of payments, or value in today’s dollars equals:

\[ V_t = z_t + \frac{1}{1 + i_t} z_{t+1} + \frac{1}{(1 + i_t)(1 + i_{t+1})} z_{t+2} + \ldots \]

• When future payments or interest rates are uncertain, then:

\[ V_t = z_t + \frac{1}{1 + i^e_t} z^e_{t+1} + \frac{1}{(1 + i_t)(1 + i_{t+1})} z^e_{t+2} + \ldots \]

• Present discounted value, or present value are another way of saying “expected present discounted value.”
Some general results

This formula has these implications:

- Present value depends positively on today’s actual payment and expected future payments.
- Present value depends negatively on current and expected future interest rates.
To focus on the effects of the sequence of payments on the present value, assume that interest rates are expected to be constant over time, then:

$$V_t = z_t + \frac{z_{t+1}^e}{1 + i} + \frac{z_{t+2}^e}{(1 + i)^2} + \cdots$$
Constant Interest Rates and Payments

When the sequence of payments is equal—called them \( z \), the present value formula simplifies to:

\[
V_t = \left[ 1 + \frac{1}{1+i} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^{n-1}} \right] z
\]

The terms in the expression in brackets represent a geometric series. Computing the sum of the series, we get:

\[
V_t = \frac{1 - \left( \frac{1}{1+i} \right)^n}{1 - \frac{1}{1+i}} \quad z
\]
References
