Lecture 13

Firms Investment

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Introduction

- In this lecture we determine how the value of the firm is affected by its decisions on investment and production, capital structure, and dividend policy.
- We also apply dynamic programing to determine the optimal investment policy for a risk-neutral firm facing quadratic capital-adjustment costs.

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Balance sheet

* productive capital	* debt (net)
	* equity

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- In practice, easier to measure the liabilities:

$$\text{value of the firm} \quad = \quad \text{debt} + \text{equity}$$

How does the value of the firm depend upon

- investment and production?
- capital structure?
- dividend policy?

Cash flows constraint

Cash flow	
Sales of output	$P_{Y_t} Y_t$
- wage cost	$-w_t N t$
- investment	$-P_{I_t} I_t$
- adjustment cost	$-P_{I_t} G\left(I_t, K_t\right)$
= net cash flow from production	$= X_t$
- dividend	$-d_t S_t$
+ new equity	$+P_{S_t}\left(S_{t+1} - S_t\right)$
+ bond emission	$+P_{B_t} B_{t+1}$
- bond repayment	$-B_t$
= net cash flow	$= 0$

The firm: main equations

- Production function

$$Y_t = F(N_t, K_t; A_t)$$

  where N is labor, K is capital, and A is productivity.

- Capital accumulation

$$K_t = \lambda K_{t-1} + I_t$$

  where λ equals 1 minus the depreciation rate.

- Cash flow constraint:

$$X_t - d_t S_t + P_{S_t}(S_{t+1} - S_t) + P_{B_t} B_{t+1} - B_t = 0$$

- Remember form Lecture 12 on asset pricing that:
$$1 = \mathbb{E}_t \left[\frac{y_{it+1} + P_{it+1}}{P_{it}} \times \frac{\beta u'(C_{t+1})}{u'(C_t)} \right] = \mathbb{E}_t[R_{it} M_t]$$

- Applying this result to the firm equity:

$$1 = \mathbb{E}_t \left[\frac{d_{t+1} + P_{St+1}}{P_{St}} M_t \right] \quad \Rightarrow P_{St} = \mathbb{E}_t \left[(d_{t+1} + P_{St+1}) M_t \right]$$

- whereas applying it to the firm debt (assuming risk-free)

$$1 = \mathbb{E}_t \left[\frac{1}{P_{Bt}} M_t \right] \quad \Rightarrow P_{Bt} = \mathbb{E}_t \left[1 M_t \right]$$

- The value of the firm (before dividends) is

$$V_t \equiv (d_t + P_{S_t}) S_t + B_t$$
$$= X_t + P_{S_t} S_{t+1} + P_{B_t} B_{t+1}$$

- Since

$$P_{St} = \mathbb{E}_t \left[(d_{t+1} + P_{St+1}) M_t \right]$$
$$P_{Bt} = \mathbb{E}_t \left[M_t \right]$$

- we get

$$V_t = X_t + S_{t+1} \mathbb{E}_t \left[(d_{t+1} + P_{St+1}) M_t \right] + B_{t+1} \mathbb{E}_t \left[M_t \right]$$
$$= X_t + \mathbb{E}_t \left\{ M_t \left[(d_{t+1} + P_{St+1}) S_{t+1} + B_{t+1} \right] \right\}$$
$$= X_t + \mathbb{E}_t \left\{ M_t V_{t+1} \right\}$$

Substitution of V_{t+1} on the V_t results in:

$$V_t = X_t + \mathbb{E}_t \{M_t V_{t+1}\}$$
$$= X_t + \mathbb{E}_t \{M_t [X_{t+1} + \mathbb{E}_{t+1} (M_{t+1} V_{t+2})]\}$$
$$= X_t + \mathbb{E}_t [M_t X_{t+1}] + \mathbb{E}_t \{M_t \mathbb{E}_{t+1} (M_{t+1} V_{t+2})\}$$
$$= X_t + \mathbb{E}_t [M_t X_{t+1}] + \mathbb{E}_t \{M_t M_{t+1} V_{t+2}\}$$

Last step follows from the Law of Iterated Expectations.

Value of the firm depends on net cash flow from production

- If we keep iterating we get:

$$V_t = \mathbb{E}_t \left[\sum_{s=t}^{\infty} M_{t,s} X_s \right] + \lim_{s \to \infty} \mathbb{E}_t \left(M_{t,s} V_s \right)$$

where

$$M_{t,s} \equiv M_t M_{t+1} \ldots M_{s-1} = \beta^{s-t} \frac{u'(c_s)}{u'(c_t)}$$

is the MRS of consumption between periods s and t.

- Because $0 < \beta < 1$, it follows that $\lim_{s \to \infty} M_{t,s} = 0$
- Therefore, the value of the firm equals the present value of the net cash flow from production:

$$V_t = \mathbb{E}_t \left[\sum_{s=t}^{\infty} M_{t,s} X_s \right]$$

Firm value	$V_t = \mathbb{E}_t \left[\sum_{s=t}^{\infty} M_{t,s} X_s \right]$

- The value of the firm equals the present discounted value of its net cash flow from production X_t.
- The value of the firm does not depend upon:
  - the debt-equity ratio
  - the dividend policy
- Notice that these results depend on some underlying assumptions:
  - no friction in capital market:
    - no private information
    - no difficulty of contract enforcement
  - no taxes

Investment by a risk-neutral firm

- For a risk-neutral firm, $M_{t,s} = \beta^{s-t}$ and

$$V_t = X_t + \beta \, \mathbb{E}_t \left(V_{t+1} \right)$$

- The Bellman equation is:

$$V_t \left(\lambda K_{t-1} \right) = \max_{N_t, I_t} \{ P_{Y_t} Y_t - w_t N_t - P_{I_t} \left[I_t + G \left(I_t, K_t \right) \right] +$$
$$+ \beta \, \mathbb{E}_t \, V_t \left(\lambda K_t \right) \}$$

subject to

$$K_t = \lambda K_{t-1} + I_t \qquad \text{and} \qquad Y_t = F \left(N_t, K_t, A_t \right)$$

Solving the problem

- The FOCs are:

$$0 = \underbrace{P_{Y_t} F_{N_t}}_{\text{Marginal product labor}} - w_t$$

$$0 = \underbrace{P_{Y_t} F_{K_t}}_{\text{Marginal product capital}} - P_{I_t} G_{K_t} - \underbrace{P_{I_t} \left(1 + G_{I_t}\right)}_{\text{Mg cost Investm.}} + \beta \lambda \, \mathbb{E}_t \left(\frac{\partial V_{t+1}}{\partial (\lambda K_t)} \right)$$

- where

$$F_N \equiv \frac{\partial F}{\partial N}, \qquad F_K \equiv \frac{\partial F}{\partial K}, \qquad G_I \equiv \frac{\partial G}{\partial I}, \qquad G_K \equiv \frac{\partial G}{\partial K}$$

- From now on we denote the marginal product of capital by MPK$_t$

- From the envelope condition we find the marginal value of capital q_t^*:

$$q_t^* \equiv \frac{\partial V_t}{\partial(\lambda K_{t-1})} = P_{Y_t} F_{K_t} - P_{I_t} G_{K_t} + \beta\lambda \,\mathbb{E}_t \left(\frac{\partial V_{t+1}}{\partial(\lambda K_t)} \right)$$

$$= \mathsf{MPK}_t + \beta\lambda \,\mathbb{E}_t \left(\frac{\partial V_{t+1}}{\partial(\lambda K_t)} \right)$$

$$= \underset{\text{Mg cost Investm.}}{P_{I_t} \left(1 + G_{I_t} \right)}$$

- the last equality follows from the second FOC

$$q_t^* \equiv \frac{\partial V_t}{\partial(\lambda K_{t-1})} = \mathsf{MPK}_t + \beta\lambda\,\mathbb{E}_t\left(\frac{\partial V_{t+1}}{\partial(\lambda K_t)}\right)$$

$$= \mathsf{MPK}_t + \beta\lambda\,\mathbb{E}_t\left(\mathsf{MPK}_{t+1} + \beta\lambda\,\mathbb{E}_{t+1}\left(\frac{\partial V_{t+2}}{\partial(\lambda K_{t+1})}\right)\right)$$

$$= \quad\vdots$$

$$= \mathbb{E}_t\left[\sum_{s=t}^{\infty}(\beta\lambda)^{s-t}\mathsf{MPK}_s\right]$$

- So far we have

$$q_t^* \equiv \frac{\partial V_t}{\partial(\lambda K_{t-1})}$$

$$= \mathbb{E}_t \left[\sum_{s=t}^{\infty} (\beta\lambda)^{s-t} \mathsf{MPK}_s \right]$$

$$= P_{I_t} \left(1 + G_{I_t} \right)$$

- Then, at the optimum investment, the marginal value of capital corresponds to the present discounted value of the marginal product of capital, and it must equal the marginal cost of investment.

Quadratic cost of investment adjustment

- Suppose that

$$G(I, K; \epsilon) = \frac{b}{2} \left(\frac{I}{K} - a - \epsilon \right)^2 K$$

- Then

$$G_{I_t}(I_t, K_t; \epsilon_t) = b \left(\frac{I_t}{K_t} - a - \epsilon \right)$$

- and optimal investment requires

$$q_t^* = P_{I_t} \left[1 + b \left(\frac{I_t}{K_t} - a - \epsilon_t \right) \right]$$

Optimal Investment	$$\frac{I_t}{K_t} = a + \frac{1}{b}\left(\frac{q_t^*}{P_{I_t}} - 1\right) + \epsilon_t$$

- This says that it is optimal to increase the growth rate of capital if the increase in firm value induced by the additional capital is higher than the market price of that capital.