Lecture 11

The firm's problem

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Introduction

- In the next lectures, we will develop models of the economy, where consumers and firms come together to exchange production factors (labor, capital) for consumption goods.
- At first, we consider *short-term* models, which assume that capital is fixed.
- We already derived the labor supply (lecture 7a); we now turn to deriving the labor demand.
- As with consumer behavior, we focus on the choices of a single, *representative firm*.

1 The representative firm

The representative firm

- The firms in this economy own productive capital and hire labor to produce consumption goods.
- The choices of the firm are determined by the available technology and by profit maximization.
- Production technology available to the firm is represented by the *production function*, which describes the technological possibilities for converting factors into outputs.

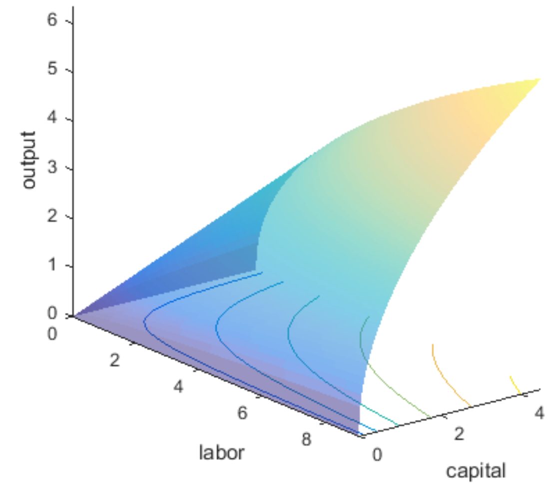
The production function

We assumed that the production function for the representative firm is described by

$$Y = zF(K, N^d)$$

where

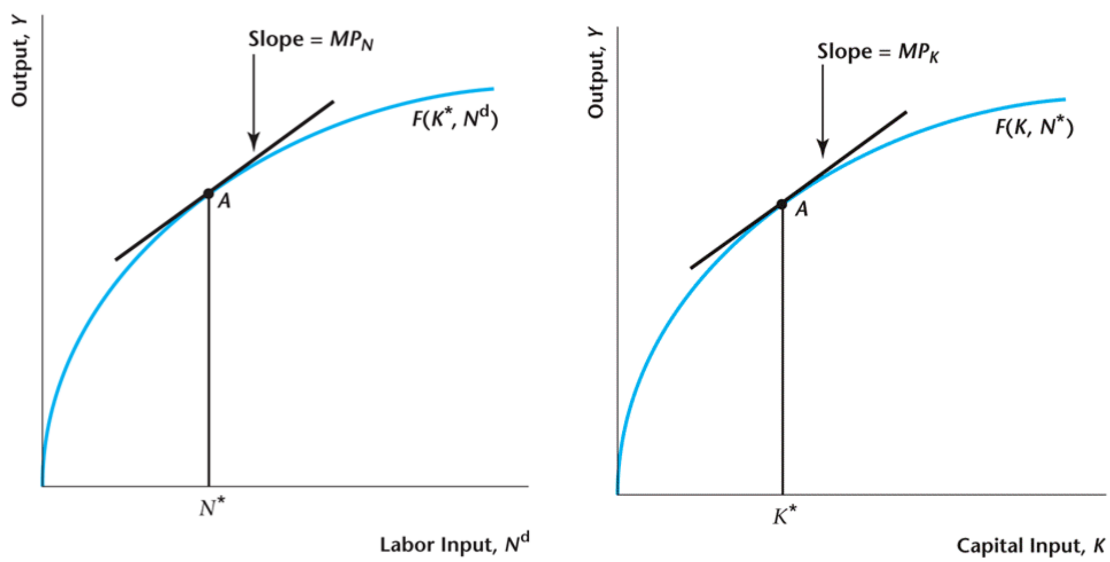
- Y is output,
- z is total factor productivity,
- K is the capital stock,
- N^d is the firm's labor input.



2 Some definitions relating the production function

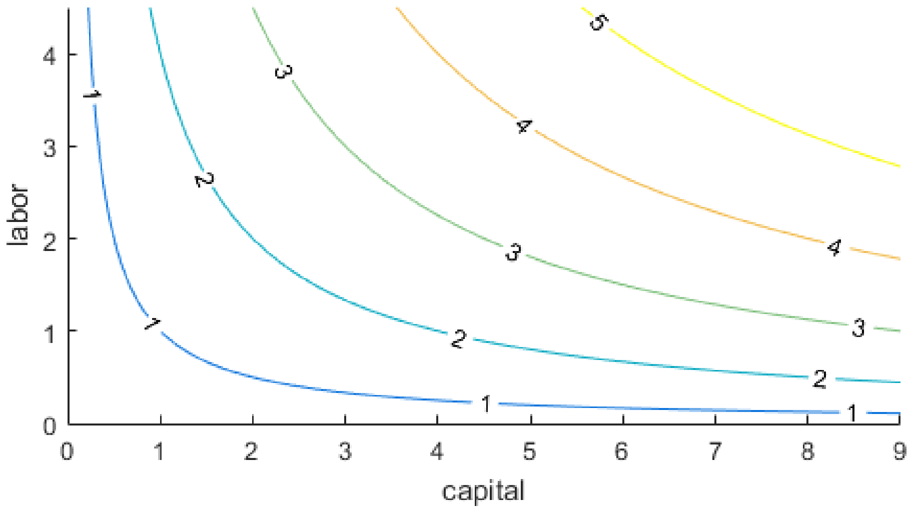
Marginal product

The *marginal product of a factor of production* is the additional output that can be produced with one additional unit of that factor input, holding constant the quantities of the other factor inputs.



Isoquants

For any fixed fixed level of output y, the set of input vectors (K, N^d) producing y units of output is called the $y-$level *isoquant*.



Marginal rate of technical substitution

- The *marginal rate of technical substitution* between capital and labor, when the current input vector is (K, N), is defined as the ratio of their marginal products:

$$\text{MRTS}_{KN} = \frac{MP_K}{MP_N} = \frac{\frac{\partial F}{\partial K}}{\frac{\partial F}{\partial N}}$$

- This measures the rate at which capital can be substituted by labor *without changing* the amount of output produced.
- It is equal to the absolute value of the slope of the isoquant through (K, N) at the point (K, N).

Elasticity of substitution

- The elasticity of substitution between labor and capital at the point (K, N) is defined as

$$\sigma_{NK} \equiv \frac{-\operatorname{dln}\left(\frac{K}{N}\right)}{\operatorname{dln}\left(\frac{F_K}{F_N}\right)} = \frac{-\Delta\%\left(\frac{K}{N}\right)}{\Delta\%\left(\frac{F_K}{F_N}\right)}$$

- When the production function is quasiconcave, the elasticity of substitution can never be negative, so $\sigma_{NK} \geq 0$

Example 1: CES production function

For a CES production function

$$Y = F(K, N) = z\left(\alpha K^\rho + \beta N^\rho\right)^{\frac{1}{\rho}}$$

the marginal products satisfy:

$$F_K = \alpha z \left(\frac{K}{Y}\right)^{\rho-1}, \quad F_N = \beta z \left(\frac{N}{Y}\right)^{\rho-1} \quad \Rightarrow \frac{F_K}{F_N} = \frac{\alpha}{\beta}\left(\frac{K}{N}\right)^{\rho-1}$$

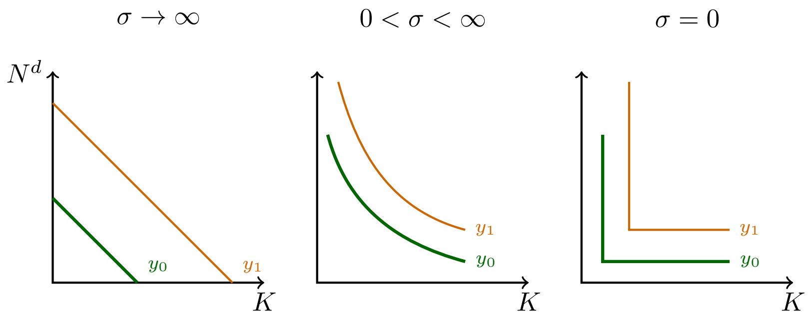
It follows that

$$\ln\left(\frac{K}{N}\right) = \frac{1}{\rho - 1}\ln\left(\frac{F_K}{F_N}\right) - \frac{1}{\rho - 1}\ln\left(\frac{\alpha}{\beta}\right)$$

Therefore, the elasticity of substitution is constant

$$\sigma_{NK} = \frac{1}{1-\rho}$$

Elasticity of substitution and isoquants



perfect substitutes

imperfect substitutes

perfect complements

3 Properties of the production function

Constant returns to scale

- The production function exhibits constant returns to scale.
- That is, if all factors are changed by a factor of x, then output changes by the same factor x:

$$zF(xK, xN^d) = xzF(K, N^d) = xY$$

Production increasing on inputs

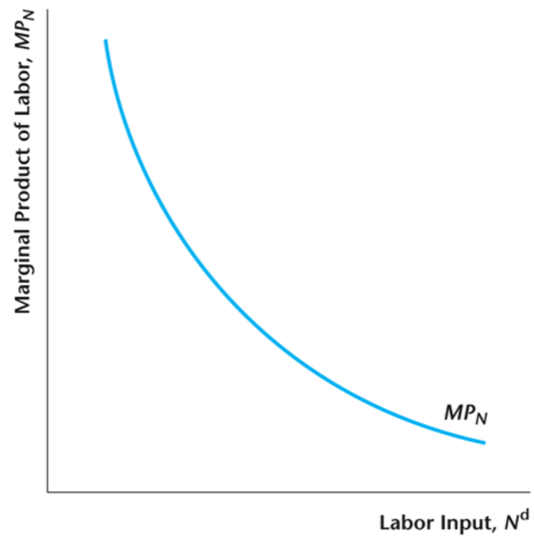
- The production function has the property that output increases when either the capital input or the labor input increases.
- In other words, the marginal products of labor and capital are both positive:

$$MP_N \equiv \frac{\partial Y}{\partial N} > 0 \qquad\qquad MP_K \equiv \frac{\partial Y}{\partial K} > 0$$

- This simply states that more inputs yield more output.

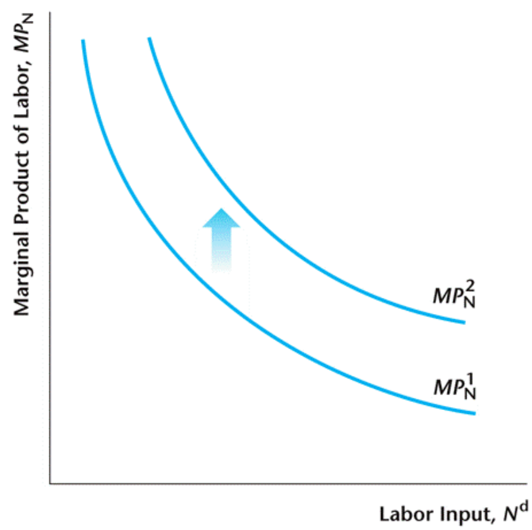
Decreasing marginal product

- The marginal product of labor decreases as the quantity of labor increases.
- The marginal product of capital decreases as the quantity of capital increases.



Marginal product of factor changes with the other factor

- The marginal product of labor increases as the quantity of the capital input increases.



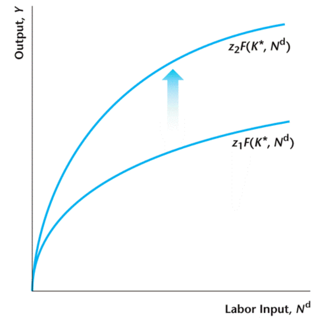
Changes in total factor productivity

- Changes in total factor productivity, z, are critical to our understanding of the causes of economic growth and business cycles.
- Productivity can change in response to:
  - technological innovation
  - weather
  - government regulations
  - changes in the relative price of energy

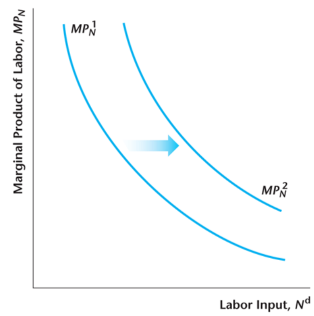
The effect of a change in total factor productivity

An increase in total factor productivity z has two important effects:

1 Because more output can be produced given capital and labor inputs when z increases, this shifts the production function up.



2 The marginal product of labor increases when z increases.



Assumptions about the production function

- The function $F(\cdot, \cdot)$ is assumed to be
  - quasi-concave,
  - strictly increasing in both arguments,
  - homogeneous of degree one or constant-returns-to-scale,
  - and twice differentiable.
- We also assume that $F_2(K, 0) = \infty$ and $F_2(K, \infty) = 0$ to guarantee that there is always an interior solution to the firm's profit-maximization problem.

4 The profit maximization problem: short term

The firm's profit-maximization problem

- The firm's profits π is the difference between revenue and labor costs in terms of consumption goods:
$$\pi = zF(K, N^d) - wN^d$$
- The firm's profit-maximization problem is to choose the labor input N^d so as to maximize profits:
$$\max_{N^d} zF(K, N^d) - wN^d \qquad (1)$$
subject to $N^d \geq 0$

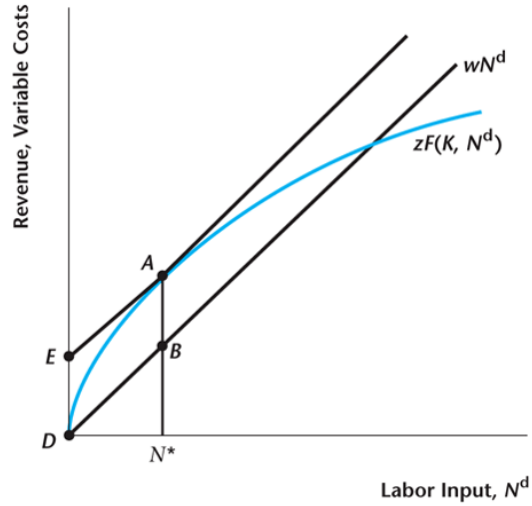
Solving the firm's problem

The restrictions on F imply that there is a unique interior solution to problem (1).

- Solution characterized by the FOC:

$$zF_2(K, N^d) = w$$

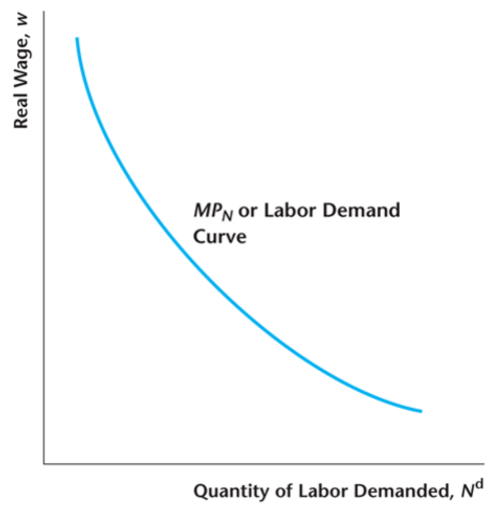
- This states that the firm hires labor until the marginal product of labor $zF_2(K, N^d)$ equals the real wage w.



Labor demand

- The representative firm's marginal product of labor schedule is the firm's demand curve for labor.

- Given a real wage w, the MP_N schedule tells us how much labor the firm needs to hire such that $MP_N = w$.



labor demand $$MP_N = zF_2(K, N^d) = w$$

Comparative statics

- We can determine the effects of changes in w, z, and K on labor demand N^d through comparative statics techniques.

- Totally differentiating labor demand equation, which determines N^d implicitly as a function of w, z, and K, we obtain

$$zF_{22}dN^d - dw + F_2dz + zF_{12}dK = 0$$

- Then, solving for the appropriate derivatives, we have

$$\frac{\partial N^d}{\partial w} = \frac{1}{zF_{22}} < 0 \qquad \text{(demand w/ negative slope)}$$

$$\frac{\partial N^d}{\partial z} = \frac{-F_2}{zF_{22}} > 0, \qquad \frac{\partial N^d}{\partial K} = \frac{-F_{12}}{F_{22}} > 0.$$

5 The profit maximization problem: long term

The firm's profit-maximization problem

- The firm's profits π is the difference between revenue and input costs in terms of consumption goods:

$$\pi = zF(K, N^d) - wN^d - rK$$

- Here we assume that the firm rents capital from representative consumer, at cost r.
- The firm's profit-maximization problem is to choose the labor input N^d and capital income K so as to maximize profits:

$$\max_{K,N^d} zF(K, N^d) - wN^d - rK \tag{2}$$

  subject to $N^d \geq 0$ and $K \geq 0$.

Solving the firm's problem

The restrictions on F imply that there is a unique interior solution to problem (2).

- Solution characterized by the FOCs:

$$\left.\begin{aligned} zF_1(K, N^d) &= r \\ zF_2(K, N^d) &= w \end{aligned}\right\} \Rightarrow \text{MRTS}_{KN} = \frac{r}{w}$$

- This states that the firm hires labor and rents capital until their marginal products equal their unit (marginal) cost.
- It also says that the marginal rate of technical substitution must be equal to the relative price of the factors.

Example 2: Optimal production with a CES function

For a CES production function

$$Y = F(K, N) = z\left(\alpha K^\rho + \beta N^\rho\right)^{\frac{1}{\rho}}$$

the MRTS is:

$$\frac{F_K}{F_N} = \frac{\alpha}{\beta}\left(\frac{K}{N}\right)^{\rho-1}$$

It follows that

$$\frac{\alpha}{\beta}\left(\frac{K}{N}\right)^{\rho-1} = \frac{r}{w}$$

The optimal capital-labor ratio satisfies

$$\frac{K}{N} = \left(\frac{\alpha w}{\beta r}\right)^{\frac{1}{1-\rho}}$$

References

Jehle, Geoffrey A. and Philip J. Reny (2001). *Advanced Microeconomic Theory*. 2nd ed. Addison Wesley. ISBN: 978-0321079169.

Williamson, Stephen D. (2014). *Macroeconomics*. 5th ed. Pearson.