Universidad de Costa Rica
EC3201 - Teoría Macroeconómica 2

Practice 6: A convex utility function

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II Semestre 2017
Last updated: October 3, 2017

A consumer problem: 2 goods

A consumer gets utility from consuming goods x and y according to the utility function:

$$U(x, y) = x^2 + y^2 \tag{1}$$

Since the prices of the goods are p_x and p_y, the budget constraint is

$$p_x x + p_y y = M \tag{2}$$

where M is available income. The consumer wants to maximize utility given his budget constraint. The Lagrangian for this optimization problem is

$$\mathcal{L}(x, y, \lambda) = x^2 + y^2 + \lambda(M - p_x x - p_y y) \tag{3}$$

where λ is the Lagrange multiplier associated with the budget constraint. The first-order condition corresponding to this Lagrangian are

$$2x = \lambda p_x \tag{4a}$$
$$2y = \lambda p_y \tag{4b}$$
$$p_x x + p_y y = M \tag{4c}$$

1. (a) Solve the system of equations (4) to obtain x^*, y^*, λ^*.
   (b) Obtain the value function $V(M, p_x, p_y) \equiv U(x^*, y^*)$
   (c) Let $U^{(i)}$ be the utility obtained from spending all available income M on good $i = x, y$. Compute $U^{(x)}$ and $U^{(y)}$.
   (d) Show that $U^{(x)} > V(M, p_x, p_y)$ and $U^{(y)} > V(M, p_x, p_y)$
   (e) The result from last question contradicts that the allocation found in part (a) is optimal, because we found two feasible bundles that generate higher utility. Explain what went wrong.

2. Repeat question 1, but assume that the utility function is

$$U(x, y) = x^2 y^2$$

   instead of $U(x, y) = x^2 + y^2$
   (a) Write down the new Lagrangian and the first-order conditions

(b) Obtain the optimal amounts x^*, y^*, λ^*

(c) Obtain the value function $V(M, p_x, p_y) \equiv U(x^*, y^*)$

(d) Compute $U^{(x)}$ and $U^{(y)}$, as defined in question 1. Show that they are less than the value function.

(e) Looks like we no longer have the contradiction mentioned in question 1e. Why not?

(f) Is the new utiliy function $U(x, y) = x^2y^2$ concave? Quasi-concave?