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Introduction
• Before we go on to build models of aggregate economic activity that can explain why business cycles exist and what, if anything, should be done about them, we must understand the key features that we observe in economic data that define a business cycle.

• In this lecture, we examine the regularities in the relationships among aggregate economic variables as they fluctuate over time.
Typically, we think of a time series as the sum of four components:

\[ \text{series} = \text{trend} + \text{cycle} + \text{seasonal} + \text{irregular} \]

After removing the trend and the seasonal components, we are left with the business cycle and the irregular component. We refer to this as deviations from trend.

For simplicity, in this lecture we neglect the irregular component and refer to deviations from trend as “cycles”.

The data we are studying in this lecture, and most data that is used in macro research and in formulating macro policy, is **seasonally adjusted**.

That is, in most macro time series, there exists a predictable seasonal component.

There are various methods for seasonally adjusting data, but the basic idea is to observe historical seasonal patterns and then take out the extra amount that we tend to see on average during a particular season.
Example 1:
Seasonally adjusted IMAE
Seasonal adjustment tends to smooth a time series with a seasonal component.
The primary defining feature of business cycles is that they are fluctuations about trend in real GDP.

We represent the trend in real GDP with a smooth curve that closely fits actual real GDP, with the trend representing that part of real GDP that can be explained by long-run growth factors.

What is left over, the deviations from trend, we take to represent business cycle activity.
Idealized business cycle
The need for stationary series

• Many modeling techniques assume that variables are stationary:
  • ARMA
  • DSGE

• To work with non-stationary series, we usually transform (filter) the original data to obtain a stationary series.

• In this lecture, we will analyze the properties of one such transformation: the HP filter.
The techniques used to separate trend from cycle are called *filters*.

There are plenty of them! For example:

- **HP**  Hodrick-Prescott
- **FOD**  First-Order Differencing
- **BN**  Beveridge-Nelson
- **UC**  Unobservable Components
- **LT**  Linear trend
- **SEGM**  Segmented trend
- **FREQ**  Frequency Domain Masking
- **MLT**  Common deterministic trend
- **MINDEX**  One-dimensional index
- **COIN**  Cointegration
The Hodrick-Prescott filter
Disaggregation of a time series

- We have a sample of $T$ observations on random variable $Y_t$:
  \[ \{y_1, y_2, \ldots, y_T\} \]
- $Y_t$ has two components: growth (trend) $s_t$ and cycle $c_t$.
  \[ y_t = s_t + c_t \]
- We assume that the trend is a smooth curve, although not necessarily a straight line.
Data trend

$y_t = S_t + C_t$
To “extract” the trend, we look for a new series

\[ \{ s_1, s_2, \ldots, s_T \} , \]

balancing two conflicting objectives:

1. the fit to the original series
2. the resulting trend must be smooth

The relative importance of these two factors is weighed with a parameter \( \lambda \).
Formally, the trend is defined by:

\[
{s^*_i}^{HP} = \arg\min_{s_1, \ldots, s_T} \left\{ \sum_{t=1}^{T} (y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1} [(s_{t+1} - s_t) - (s_t - s_{t-1})]^2 \right\}
\]

\[
= \arg\min_{s_1, \ldots, s_T} \left\{ \sum_{t=1}^{T} (y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1} (s_{t+1} - 2s_t + s_{t-1})^2 \right\}
\]
A little trick from linear algebra

Let’s define these matrices

\[ Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix}, \quad S = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_T \end{bmatrix} \]

\[ A_{T-2 \times T} = \begin{bmatrix} 1 & -2 & 1 & 0 & \ldots & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & \ldots & 0 & 0 & 0 & 0 \\ \vdots \\ 0 & 0 & 0 & 0 & \ldots & 0 & 1 & -2 & 1 \end{bmatrix} \]
Rewriting the optimization problem

\[ s_{i}^{HP} = \arg\min_{s_{1}, \ldots, s_{T}} \left\{ \sum_{t=1}^{T} (y_{t} - s_{t})^2 + \lambda \sum_{t=2}^{T-1} (s_{t+1} - 2s_{t} + s_{t-1})^2 \right\} \]

\[ = \arg\min_{S} \left\{ (Y - S)'(Y - S) + \lambda (AS)'(AS) \right\} \]

\[ = \arg\min_{S} \left\{ Y'Y - 2Y'S + S'(I + \lambda AA')S \right\} \]
Solving the problem

• Taking the FOC

\[ S^{HP} = \arg\min_S \left\{ Y'Y - 2Y'S + S'(I + \lambda A' A)S \right\} \]

\[ \Rightarrow -2Y + 2 (I + \lambda A' A) S = 0 \]

• Then, the HP filter is

\[ S^{HP} = (I + \lambda A' A)^{-1} Y \] (trend)

\[ C^{HP} \equiv Y - S^{HP} = \left[ I - (I + \lambda A' A)^{-1} \right] Y \] (cycle)
Choosing $\lambda$

- The result of filtering is very sensitive to the choice of $\lambda$
- As a rule of thumb, $\lambda$ is chosen depending on frequency of data.
  - Annual $\Rightarrow 100$
  - Quarterly $\Rightarrow 1600$
  - Monthly $\Rightarrow 14400$
Example 2:
Filtered series when $\lambda = 1600$
USA real GDP, 2009 dollars

billions of 2009 US$


% deviation from trend


GDP
GDP_trend
GDP_cycle
USA real consumption, 2009 dollars

![Graph showing USA real consumption in 2009 dollars from 1949 to 2009. The graph plots billions of 2009 US$ on the y-axis and years from 1949 to 2009 on the x-axis. Two lines are present: one for the actual consumption (C) and another for the trend (C_trend). The consumption shows a general increase over the years, with fluctuations representing the cycle (C_cycle).]
Regularities in GDP fluctuations
• **Business cycles are quite irregular**: the changes in real GDP are unpredictable; it’s very difficult to predict the timing of a business cycle upturn or downturn.

• **Business cycles are quite regular**, however, in terms of comovements: macroeconomic variables move together in highly predictable ways.
Real GDP cycles from 1947 to 2012
Persistent but irregular

- Real GDP cycles are persistent:
  - when real GDP is above trend, it tends to stay above trend
  - when it is below trend, it tends to stay below trend.

- Real GDP cycles are quite irregular.
  1. The time series of real GDP cycles is quite choppy.
  2. There is no regularity in the amplitude of fluctuations in real GDP about trend. Some of the peaks and troughs represent large deviations from trend, whereas other peaks and troughs represent small deviations from trend.
  3. There is no regularity in the frequency of fluctuations in real GDP about trend. The length of time between peaks and troughs in real GDP varies considerably.
Forecasting implications

- Because real GDP cycles are persistent, short-term forecasting is relatively easy.
- But because they are irregular longer-term forecasting is difficult:
  - the choppiness of fluctuations in real GDP makes these fluctuations hard to predict
  - the lack of regularity in the amplitude and frequency of fluctuations implies that it is difficult to predict the severity and length of recessions and booms.
Comovement
While real GDP fluctuations are irregular, macro variables fluctuate together in strongly regular patterns. We refer to these patterns in fluctuations as comovement. Macro variables are measured as time series; for example, real GDP is measured in a series of quarterly observations over time. When we examine comovements in macro time series, typically we look at these time series two at a time. A good starting point is to plot the data.
Plotting in time domain

Positive Correlation Between $x$ and $y$

Negative Correlation Between $x$ and $y$
Plotting a scatter plot

(a) Positive Correlation Between $y$ and $x$

(b) Negative Correlation Between $y$ and $x$

(c) Zero Correlation Between $y$ and $x$
Primary interest: how an individual macro variable comoves with real GDP.

An economic variable is said to be:

- **procyclical** if its cycles are positively correlated with the real GDP cycles,
- **countercyclical** if its cycles are negatively correlated with the real GDP cycles,
- **acyclical** if it is neither procyclical nor countercyclical
Example 3:

Imports comovement
Imports and GDP are clearly positively correlated, so imports are procyclical.
We again observe the positive correlation between imports and GDP, as a positively sloped straight line would best fit the scatter plot. Again, imports are procyclical.
Leading and lagging variables

An important element of comovement is the leading and lagging relationships that exist in macroeconomic data.

- A **leading variable** is a macro variable that tends to aid in predicting the future path of real GDP.
- If real GDP helps to predict the future path of a particular macroeconomic variable, then that variable is said to be a **lagging variable**.
- A **coincident variable** is one which neither leads nor lags real GDP.
Idealized cycles in real GDP and two variables

(a) $x$ is a leading variable

(b) $y$ is a lagging variable
Some leading indicators

• A knowledge of the regularities in leading relationships among economic variables can be very useful in macro forecasting and policymaking.

• Typically, macro variables that efficiently summarize available information about future macro activity are potentially useful in predicting the future path of real GDP.

• For example,
  • the stock market
  • the number of housing starts
Example 4:
Housing starts as a leading indicator
Percentage deviations in housing starts are divided by 10 so we can see the comovement better. Housing starts clearly lead real GDP (note the timing of turning points in particular).
Consumption

It’s procyclical, coincident, and less variable than GDP.
Investment

It’s procyclical, coincident, and more variable than GDP.
Price level

It’s countercyclical, coincident, and less variable than real GDP
Money supply

It’s procyclical and leading variable, and it is less variable than real GDP.
Employment

It’s procyclical, it is a lagging variable, and it is less variable than real GDP.
Average labor productivity is procyclical and coincident, and it is less variable than is real GDP.
Summary of results
### Correlation coefficients and variability of cycles

<table>
<thead>
<tr>
<th></th>
<th>Correlation Coefficient</th>
<th>Standard Deviation*</th>
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</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>0.78</td>
<td>76.6</td>
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<tr>
<td>Investment</td>
<td>0.85</td>
<td>489.9</td>
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<td>Money Supply</td>
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<td>Employment</td>
<td>0.80</td>
<td>63.0</td>
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<tr>
<td>Avg. Labor Productivity</td>
<td>0.80</td>
<td>62.4</td>
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</table>

* % of standard deviation of GDP
## Summary of business cycle facts

<table>
<thead>
<tr>
<th>Metric</th>
<th>Cyclicality</th>
<th>Lead/Lag</th>
<th>Variation Relative to GDP</th>
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<tbody>
<tr>
<td>Consumption</td>
<td>Procyclical</td>
<td>Coincident</td>
<td>Smaller</td>
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<tr>
<td>Investment</td>
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<td>Coincident</td>
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<td>Employment</td>
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<td>Real Wage</td>
<td>Procyclical</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Avg. Labor Productivity</td>
<td>Procyclical</td>
<td>Coincident</td>
<td>Smaller</td>
</tr>
</tbody>
</table>
Some warnings
1. The practice of solely employing the HP1600 filter in compiling business cycle statistics is problematic.

2. The idea that there is a single set of facts which is more or less robust to the exact definition of business cycle is misleading.

3. The empirical characterization of the B.C. obtained with multivariate detrending methods is different from the one obtained with univariate procedures.

4. The practice of building theoretical models whose numerical versions quantitatively match one set of regularities obtained with a particular concept of cyclical fluctuation warrants a careful reconsideration.
# U.S. Business Cycle: standard deviations

<table>
<thead>
<tr>
<th>Filter</th>
<th>GNP</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Real wage</th>
<th>Productivity</th>
<th>Capital</th>
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<td>UC</td>
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<td>FREQ1</td>
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<td>FREQ2</td>
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<td>MINDX</td>
<td>3.47</td>
<td>0.98</td>
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<td>COIN</td>
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<td>1.68</td>
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<td>1.30</td>
</tr>
</tbody>
</table>

(absolute for GNP, all others relative to GNP)
References

